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POSITIONING MODEL OF TARGET SHADOW BASED ON GENETIC ALGORITHM

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ABSTRACT

As an important aspect of video data analysis technology, object shadow positioning technology based on video is a method to determine the shooting location and time of video by analyzing the solar shadow changes of objects in video. This paper analyzes and discusses the change of shadow length. Firstly, the concept of solar height angle is introduced according to the principle of shadow generation. By analyzing the determinants of solar height angle, the functional relationship between shadow length and various parameters is established, and then the genetic algorithm is used to establish the targeted positioning model of sun shadow. Finally, test the model with actual data.

Keywords: Genetic Algorithm, Targeted Location Model, Shadow, Solar Altitude Angle, Declination

1. INTRODUCTION

In the history of social development, the observation of solar shadow is a universal method of timing and positioning. The emerging video capture technology provides a large amount of continuous correlation data for the sun shadow timing and positioning technology, making it possible to establish quantitative analysis model. Sun shadow timing positioning technology is

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gradually being applied to more and more fields, such as criminal investigation, military operations, etc. Therefore, it is necessary to establish a set of accurate and effective models.

In the field of solar shadow positioning modeling, many domestic and foreign scholars have made a lot of research on the selection of modeling methods and the optimization of models. The current mainstream mathematical modeling methods include Monte Carlo algorithm, data fitting, grid algorithm, exhaustive algorithm, neural network analysis method, genetic algorithm, mathematical geometry method, etc. At present, in the research field of sun shadow localization problem, the main algorithms used are enumeration method and data fitting method, the most common of which is least square method.

Based on the principle of ancient sundial, Wu Junbin et al. obtained reliable positioning results by using least square method through Java program simulation and verification (Wu, Wu&Wu, 2016). Tang Shichao et al. also selected the least squares fitting, adding trigonometric functions and other methods, considering various factors such as the direct solar latitude angle and the latitude of the observation point (Tang, Jiang, 2016). Also based on the nonlinear least-square fitting optimization model, there are many other scholars, whose research mainly adopts the least square method, synthesizes the related knowledge of astronomy and geography, and partially combines other mathematical methods and modeling methods to obtain the results of accuracy within an acceptable range.

Other methods are also widely used. According to the declination formula and the solar height angle formula, Li Yuanyuan, the director of the Wuhan Institute of Mathematics and Computer Science, obtained the precise curve of the shadow change. She used enumeration method to model, which eliminated camera calibration step in general visual measurement (Zhang, Li&Zhang et al., 2016). Ma Yuxue analyzes the trend of the sun shadow length data and obtains the quadratic function fitting equation of shadow length and time and determines the longitude of the shooting location according to the time point with shortest length (Ma, Linghu & Yang et al., 2015). Shen Shuyun et al. also used the Analemmatic sundial model, combined with the quadratic fitting and the law of sun shadow length variation, and used Matlab programming as the tool to provide the sun shadow positioning (Shen, Wang &Guo, 2017). Recently, the mathematical modeling research of sun shadow positioning continues to develop, and many new methods have been tried based on mainstream models. The introduction of the new model has improved the accuracy of mathematical model of the sun's shadow location, but there are still improvements.

For example, Peng Sifan of China University of Geosciences adopted the modeling method of mirror image principle to restore two-dimensional data into three-dimensional data and

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established a valid model. However, this model ignores the influence of atmospheric radiation and has certain deviations (Peng, Qi, 2016). Reasonable model results can also be obtained by direct analysis using mathematical geometry methods. For example, Yu Runze uses mathematical geometry methods, combined with least squares method, fitting method, cosine theorem, coordinate transformation of spatial perspective and other processing methods to figured out the time of shooting objects (Yu, 2017).

In this paper, we analyze and discuss the change of shadow length. Firstly, we introduce the concept of solar elevation angle according to the principle of shadow generation. By analyzing the determinants of solar elevation angle, we establish the functional relationship between shadow length and various parameters. Then the genetic algorithm is used to establish the targeted positioning model of the shadow, and the encoded data is input into Matlab for solution. Finally, the model is tested with the actual data.

2. SUN SHADOW LENGTH VARIATION MODEL

The reflection is the shadow generated by the sunlight when it encounters an opaque object in the process of propagation. Considering that the change of the shadow length of the object is related to the position of the sun, the sun height angle is introduced to solve the shadow length. The relative position of the location of the sun and observation point determines the solar elevation angle.

Since the relative position of the sun's location point and the observation point is determined by three factors of the point's, which are latitude, season (month, day) and time, the relationship between reflection and the variables such as latitude, season, time, and solar height angle can be used to study the variation of the length of the shadow.

2.1 Introduction of the concept of the solar elevation angle

The solar elevation angle is the angle between the direct sunlight and the ground plane, which varies with local time and declination (Wang, Mi & Deng, 2007) (Zheng, Lin & Liu, 2010). The calculation formula is as follows:

$$\sin\theta = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos w$$

Where, θ represents the solar elevation angle;

φ represents the geographic latitude of the observation site;

δ represents the declination angle;

w represents the time angle.

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2.2 Introduction of the concept of declination

The declination angle is the angle between the plane of the earth's equator and the connection between the earth's center and the sun's center, also known as the direct incidence angle of the sun (Chen, Zhen, 2011). The declination angle is an illusion caused by the earth's orbit around the sun. Its size changes with time and has different values with different points on the earth's orbit. By consulting the literature, it is found that the declination angles in the spring equinox, the summer solstice, the autumnal equinox, and the winter solstice are respectively 0°, $23^\circ 26'$, 0° and $-23^\circ 26'$, so the declination angle at any point in the year can be calculated by setting the time interval. The calculation process is as follows:

Step 1 Determine the time interval in which the observation date is located

The year is divided into four time intervals, and the declination angle of each interval is as shown in Table 1:

Table 1. Relationship between the declination angle and the measurement date

Interval	Date	Declination range
Interval 1	3. 20-6. 20	$0^\circ < \delta < 23^\circ 26'$
Interval 2	6. 21-9. 22	$0^\circ < \delta < 23^\circ 26'$
Interval 3	9. 23-12. 21	$-23^\circ 26' < \delta < 0^\circ$
Interval 4	12. 22-3. 19	$-23^\circ 26' < \delta < 0^\circ$

Step 2

Calculation of the declination angle of the observation date

If the measurement date is within interval 1, the

$$\delta = \frac{23^\circ 26'}{93} \times N_0$$

Where δ represents the declination angle;

N_0 represents the time interval between measurement date and vernal equinox.

If the measurement date is within interval 2, then:

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$$\delta = 23^{\circ}26' - (\frac{23^{\circ}26'}{93} \times N_0)$$

Where δ represents the declination angle;

N_0 represents the time interval between measurement date and summer solstice.

If the measurement date is within interval 3, then:

$$\delta = -\frac{23^{\circ}26'}{90} \times N_0$$

Where δ represents the declination angle;

N_0 represents the time interval between measurement date and autumnal equinox.

If the measurement date is within interval 4, then:

$$\delta = -23^{\circ}26' + \frac{23^{\circ}26'}{90} \times N_0$$

Where δ represents the declination angle;

N_0 represents the time interval between measurement date and winter solstice.

2.3 Introduction of the concept of time angle

The earth rotates 360° a day, and 15° an hour, which is called the time angle. The time angle formula is calculated as follows:

$$w = 15^{\circ} \times (ST - 12)$$

$$ST = T - n$$

$$n = \frac{E - 120^{\circ}}{15^{\circ}}$$

Where, w represents the time angle;

ST represents the true solar time;

T represents the Beijing time;

n represents the time difference;

E represents the test point longitude.

2.4 Calculation of shadow length

The shadow length of the observation point changes with time, and the changing process is shown in Figure 2.

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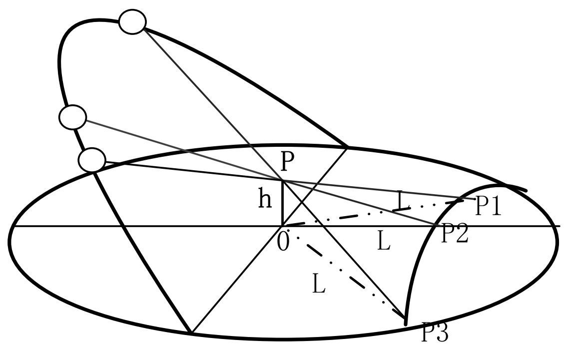

Fig. 2. Variation of shadow length with solar altitude Angle

The calculation formula is as follows:

$$l = \frac{h}{\tan\theta}$$

Where h represents the height of the object;

θ represents the Solar elevation angle.

In summary, the relationship between the shadow length and each parameter is derived from equations shown above. When the observation date is in interval 1, the calculation formula of shadow length is as follows:

$$\begin{cases} l = \dfrac{h}{\tan\theta} \\ \theta = \mathrm{arc}(\sin\varphi\sin(\dfrac{23^{\circ}26'}{93}\times N_0) + \cos\varphi\cos(\dfrac{23^{\circ}26'}{93}\times N_0)\cos(15^{\circ}\times(T - \dfrac{E-120^{\circ}}{15^{\circ}} - 12)) \end{cases}$$

When observing date’s in section 2, the calculation formula of the length of the shadow is as follow:

$$\begin{cases} l = \dfrac{h}{\tan\theta} \\ \theta = \mathrm{arc}(\sin\varphi\sin(-\dfrac{23^{\circ}26'}{93}\times N_0) + \cos\varphi\cos(-\dfrac{23^{\circ}26'}{93}\times N_0)\cos(15^{\circ}\times(T - \dfrac{E-120^{\circ}}{15^{\circ}} - 12)) \end{cases}$$

When observing date’s in section 3, the calculation formula of the length of the shadow is as follow:

$$\begin{cases} l = \dfrac{h}{\tan\theta} \\ \theta = \mathrm{arc}(\sin\varphi\sin(-\dfrac{23^{\circ}26'}{90}\times N_0) + \cos\varphi\cos(-\dfrac{23^{\circ}26'}{90}\times N_0)\cos(15^{\circ}\times(T - \dfrac{E-120^{\circ}}{15^{\circ}} - 12)) \end{cases}$$

When observing date’s in section 4, the calculation formula of the length of the shadow is as follow:

$$\begin{cases} l = \dfrac{h}{\tan\theta} \\ \theta = \arc(\sin\varphi\sin(-23^{\circ}26' + \dfrac{23^{\circ}26'}{90}\times N_0) + \cos\varphi\cos(-23^{\circ}26' + \dfrac{23^{\circ}26'}{90}\times N_0)\cos(15^{\circ}\times(T - \dfrac{E-120^{\circ}}{15^{\circ}} - 12)) \end{cases}$$

2.5 Length of shadow and change rules of each parameter

By analyzing the calculation formula of the length of the shadow, we can see the changes of the length of shadow and the length of straight bar, longitude, latitude as well as the observing location are related to time. Regard the length of straight bar, longitude, latitude, observing date and time of observing location as independent variable respectively, consider sub length as dependent variable, then draw the figure, we can conclude the change rules that shadow length varies with each parameter by observing the figure.

Select Guangzhou, Guangdong Province (longitude: $113^{\circ}23'$, latitude: $23^{\circ}16'$), observing date is November 10, 2015, time is 10 a.m., the image of shadow length and straight bar length is shown in figure.3 below:

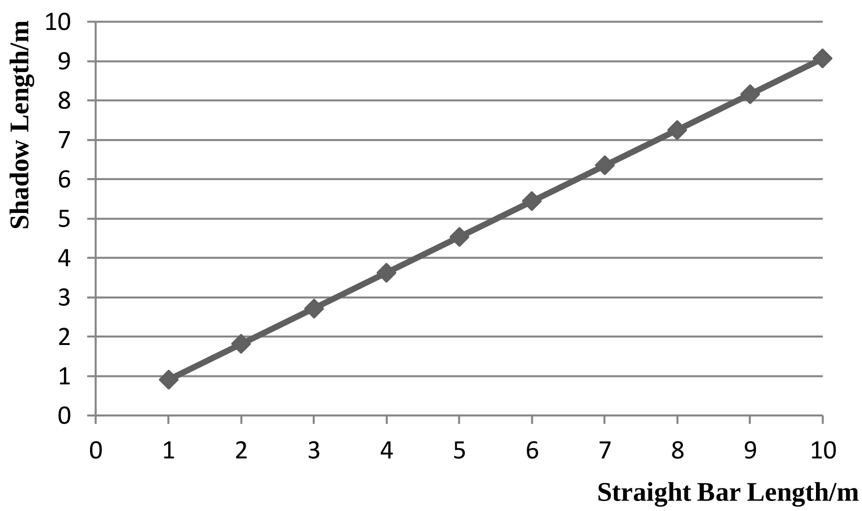

Fig. 3. Relationship between shadow length and straight bar length

Analyzing figure 3, horizontal axis represents straight bar length, vertical axis represents shadow length, when remaining longitude, latitude of observing location, observing date and time as well as other variable constant, straight bar length has positive correlation with shadow length, and such correlation approximates linear correlation.

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Select Guangzhou, Guangdong Province (longitude: $113^{\circ}23'$, latitude: $23^{\circ}16'$), observing date is November 10, 2015, the length of straight bar is 3 meters, image of shadow length and straight bar length is shown as figure 4 below:

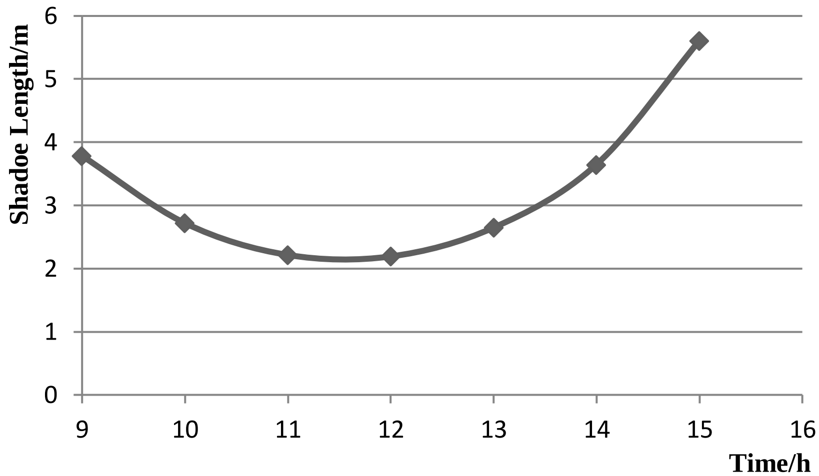

Fig. 4. Relationship between shadow length and time

Analyzing figure 4, horizontal axis represents time, vertical axis represents shadow length, when remaining observing location, observing time as well as other variables constant, between morning and noon, shadow length becomes shorter gradually, between noon and afternoon, shadow length becomes longer gradually, and when the time is closer to high noon, the smaller the variation trend is.

Select Guangzhou, Guangdong Province (longitude: $113^{\circ}23'$, latitude: $23^{\circ}16'$), the length of straight bar is 3 meters, consider 10 a.m. each morning as observing time, October 1, 2015 to December 10, 2015 as observing dates, the image of shadow length varies with date is shown as figure 5 below:

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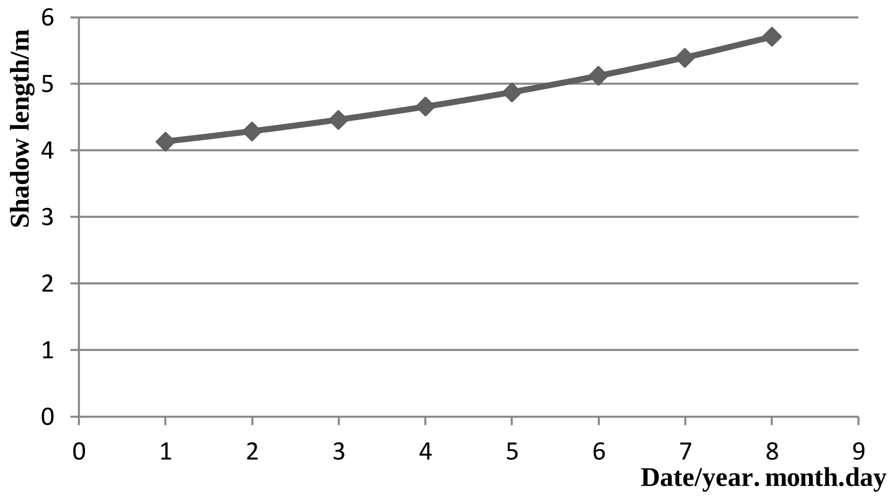

Fig. 5. Relationship between shadow length and date

Analyzing figure 5, horizontal axis represents changes of dates, vertical axis represents shadow length, when remaining observing location, time as well as other variables constant, the length of shadow grows longer with the increase of dates.

Select longitude as $113^{\circ}23'$, straight bar length as 3 meters, observing date is November 10, 2015, 10:00 a.m., measure the changes of straight bar shadow length in different latitude, obtain image as figure 6 shown below:

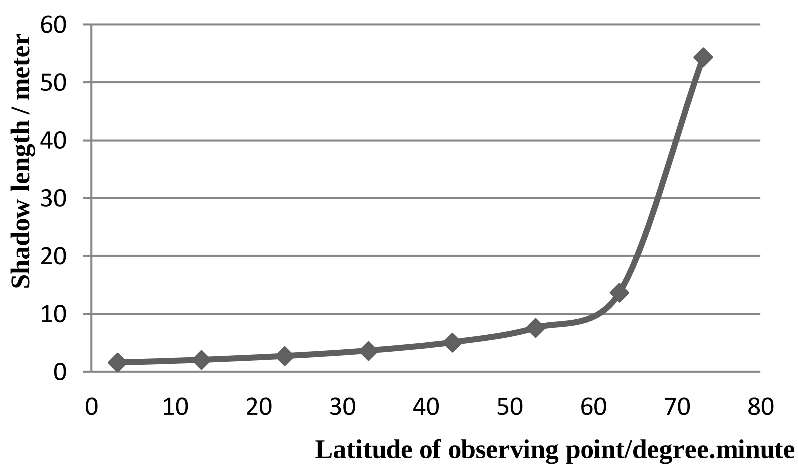

Fig. 6. Relationship between shadow length and latitude of observing locations

Analyzing figure 6, horizontal axis represents latitude of observing locations, vertical axis represents shadow length, when remaining longitude of locations, observing date and time constant, shadow length grows longer with the increase of latitude, and the increasing trend grow larger gradually.

Select latitude as $23^{\circ}16'$, the length of straight bar as 3 meters, observing date as November 10, 2015, 10:00 a.m., measure the changes of straight bar shadow length in different longitude, obtain image as figure 7 shown below:

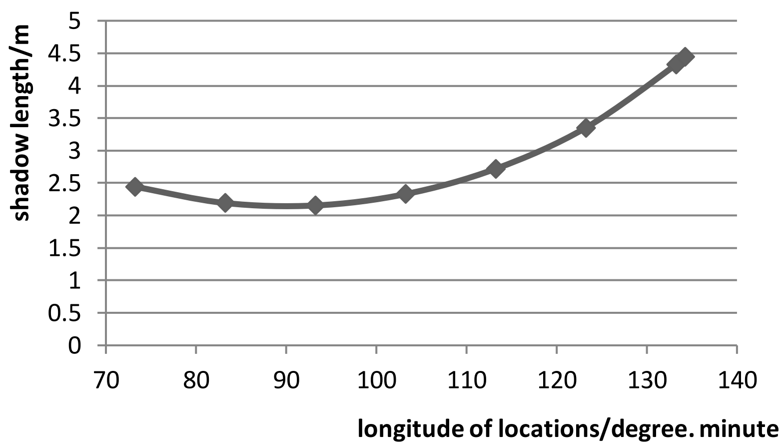

Fig. 7. Relationship between shadow length and longitude of observing locations

Analyzing figure 7, horizontal axis represents longitude of observing locations, vertical axis represents shadow length, when remaining latitude of locations, observing date, time and other variables constant, shadow length shortens with the increase of longitude firstly, then grows longer gradually, the variation trend is relatively large when the longitude is large.

3. LOCALIZATION MODEL BASED ON GENETIC ALGORITHM AND SHADOW OF THE SUN

3.1 Determination of the world's land area

In order to determinate the value range of longitude and latitude, in the first place exclude the sea area through world map and obtain latitude range that's between $-40^{\circ} \sim 80^{\circ}$ (south latitude represents negative, north latitude represents positive), then utilize the principles of terminator , regard six o'clock everyday as morning line, eighteen o'clock as dusk line, then use formula shown below to obtain terminator which corresponds to 21 time points:

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$$\text{Morning line} = 120° - (\text{Beijing time - 6}) \times 15°$$
$$\text{Dusk line} = 120° + (18 + \text{Beijing time}) \times 15°$$

Among those, Beijing time is in decimal form.

Conclude the shadow length is greater than zero by analyzing the date of Title annex two, so the range of longitude means determining all districts which are in the daytime as well as in the Beijing time range given by title, then select the maximized value in morning lines' data and the minimized value in dusk lines' data from 21 groups of terminators' data as longitude range, the result is $-10.5° \sim 160.5°$.

3.2 Localization in shadow of the sun based on genetic algorithm (Zhuo, 2014) (Bian , Mi, 2010)

Step 1 Coding

Set range of longitude as $-10.5° \sim 160.5°$, range of latitude as $-40° \sim 80°$, range of straight bar height as $0 \sim 5m$, accuracy is Four decimal places, do binary coding on variables successively:

$$2^{x_1-1} < [160.5-(-10.5)] \times 10^4] \le 2^{x_1} - 1$$
$$2^{x_2-1} < [80-40] \times 10^4] \le 2^{x_2} - 1$$
$$2^{x_3-1} < [5-0] \times 10^4] \le 2^{x_3} - 1$$

Among those, x_1 represents chromosome number in Longitude binary form, x_2 represents chromosome number in latitude binary form, x_3 represents chromosome number in straight bar height binary form.

To solve to get $x_1 = 18$, $x_2 = 18$, $x_3 = 12$, and the digits of chromosome sequence is the sum of these three in the algorithm, those are 58 digits, number 1 to 18 are longitude, number 19 to 36 are latitude, number 37 to 58 are height of straight bar. Set initial population quantity as 300, generate 300 chromosome string in random, then transfer those 300 random chromosome string to three variables' dotted-decimal format one by one shown as below (exemplify the calculation of longitude):

$$y_1 = -10.5 + Y_1 \times \frac{160.5-(-10.5)}{2^{x_1}-1}$$

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Among those, y_1 represents longitude's Decimal value, Y_1 represents decimal result of the longitude part of chromosome, x_1 represents chromosome number of the longitude part of chromosome, then obtaining decimal result of latitude and straight bar height in the similar way.

Step 2 calculation of fitness

Set objective function as fitness function, objective function is

$$y = -10 \times \sum_{n=1}^{21} (\sin(\mathrm{h}_i) - \frac{y_{3i}}{\sqrt{y_{3i}{}^2 + s_i{}^2}})^2$$

$$\sin(h_i) = \sin y_{2i} \cdot \sin\delta + \cos y_{2i} \cdot \cos\delta \cdot \cos((t_i + \frac{(y_{1i} - 120)}{15} - 12) \times 15)$$

y represents the result of objective function, h_i represents solar altitude corresponding to number i group's data, y_{1i} represents longitude corresponding to number i group's data , y_{2i} represents latitude corresponding to number i group's data, y_{3i} represents the height of straight bar corresponding to number i group's data, t_i represents Beijing time of number i group's data, δ represents declination(obtained from the solution of question one), and the units of angle, longitude, latitude, declination all are radian.

The result of objective function, which means quadratic sum of two solar altitude sin value's difference value, multiplies -10, due to the need of getting minimized value of function, multiplies the result by negative 1, in the meantime magnifying the result to make it more apparent.

Step 3 selection and copulation

According to each individual's fitness, calculate the probability of selecting each individual, the calculating function is as follows:

$$\text{Number } i \text{ individual's selection probability} = \frac{\text{Number } i \text{ individual's fitness}}{\text{Sum of fitness of all individuals}}$$

Then, according to the probability, two individuals in the population are selected to form a pair, and the number of individuals is 300, so a total of 150 groups are formed, and then the mating probability is set to 0.90. If the mating is determined according to the mating probability, a

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crossover point, which is a natural number from 1 to 58, is selected for chromosome segment exchange.

Step 4 Mutation

For each individual, randomly select a position in the chromosome string as the chromosomal mutation point, and then set the mutation probability to 0.09. If the mutation occurs, to chromosome which is represented by binary, is 1 to 0 or 0 to 1.

Step 5 Iteration and recording

Calculate the maximum value of the objective function and the corresponding longitude, latitude, and straight bar heights obtained in this generation, and record them. Then repeat Step 2 to Step 5 according to the set maximum algebra 500 and compare the results with the past. The optimal value is the maximum value of the objective function in the 500 generation.

3.3 Location analysis based on measured data

Using *Matlab* programming, and drawing the fitness curve, to reduce the contingency, the program is run 100 times, the solution of 100 optimal values and the corresponding values of the three variables are solved, then the combination of the best value result greater than or equal to -0. 003. is selected as the result and take into account the geographical location points falling in the ocean domain in the rejection result, we can get 9 optimal results, and plot the coordinates of the result in the world map. As can be seen in Table 8 and Figure 10:

Table 8. Observation position and length

Location	Longitude	Latitude	Bar heights	Optimal function value
Myanmar	96°15 " 57′	19°3 " 31′	3. 0561	-0. 0028
Myanmar	95°38 " 45′	23°42 " 37′	2. 7957	-0. 0015
Myanmar	93°54 " 39′	19°0 " 40′	3. 3157	-0. 0026
Laos	101°40 " 25′	22°1 " 46′	2. 3909	-2.94×10^{-4}
Hainan	110°27 "	18°51 " 52′	1. 8377	-0. 001
Vietnam	103°16 " 59′	21°20 " 48′	2. 2678	-0. 0012
India	88°55 " 15′	21°52 " 52′	3. 7625	-9.56×10^{-4}
India	92°2 " 33′	24°2 " 20′	3. 1673	-6.85×10^{-4}
Yunnan	101°3 " 26′	23°46 " 52′	2. 3633	-0. 0017

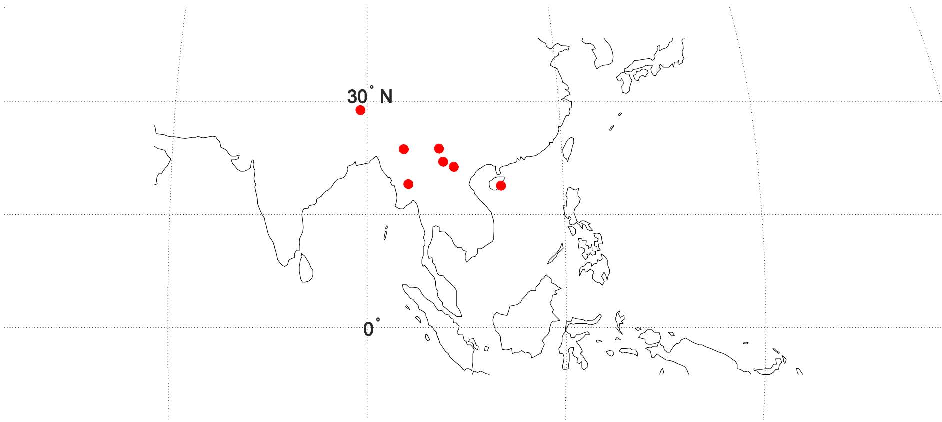

Fig. 9. Distribution of observation sites

From the results of Table 8 and Figure 9, it is known that the shadow of the sun is mainly concentrated in Myanmar, Laos, India (near Myanmar), Vietnam and Yunnan, mainly in Southeast Asia and southwestern China. From Figure 9, the analysis of the results of the use of genetic algorithms for the positioning of the sun's shadows, the regional concentration rate is higher, therefore, the accuracy of the results of the model can be considered higher.

At the same time, the fitness curve after the operation of the genetic algorithm is given. Due to the space, only the positional result is given as the fitness curve of Laos.

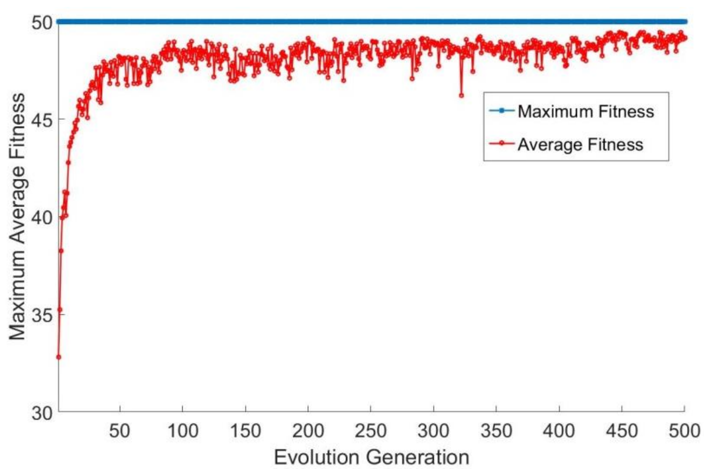

Fig. 10. Evolutionary Algebra and Average Fitness

As can be seen in the evolution process, the average fitness and maximum fitness of the population have mutually similar patterns on the curve, indicating that the algorithm converges

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smoothly and there is no oscillation, indicating that the accuracy of the model results is very high.

4. CONCLUSIONS

This dissertation analyzed and discussed the changes in shadow length, based on the data of the object's shadow, and using the genetic algorithm to construct the target point location model. This dissertation first introduced the concept of solar elevation angle according to the principle of shadow generation, built up a functional relationship between the shadow length and each parameter is established through analyzing the determinants of the solar elevation angle. Then, used genetic algorithm to establish the positioning model of the shadow, and test the model with the actual. Finally, tested the stability and accuracy of the model based on the measured data. The analysis results show that the model has good accuracy and robustness.

This dissertation used natural geography and genetic algorithm and built a positioning model based on the target object's shadow data to the target object. This method is applied to the location of the shooting location in video analysis. It is worth noting that the target point location model based on physical geography and genetic algorithm has good versatility, because the model can be applied to the field's question through video analysis by changing the parameters of the algorithm and making some adjustments in the details.

In future research, we will further improve our model research. For example, the integrity and scientificity of the positioning model will be further improved, making the positioning accuracy more reasonable. In addition, we will try to design different parameters of the genetic algorithm so that the simulation can be more practical.

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