

OPTIMIZATION ANALYSIS OF CT SYSTEM IMAGING

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ABSTRACT

Aiming at the problem of how to reconstruct an image based on the absorbed data from the unknown medium using the CT system with known calibration parameters, this paper proposed the Optimized filtered back-projection model of Image Denoising. This model first uses the Ramp-Lak filtered back-projection algorithm to reconstruct the image of the unknown medium to obtain the position and geometry of the medium. Then, considering the limitations of the single filtered back-projection algorithm, Shepp-Logan filtered back-projection algorithm and Lewitt filtered back-projection algorithm was used separately to reconstruct the CT image. Then the gray-scale variance function is used to calculate the sharpness of the above three groups of images. The image used the Ramp-Lak filtered back-projection algorithm got the highest score, indicating that the resolution is the best. Then, because the noise signal generated during the back-projection of the image will affect the sharpness of the image, the Wiener algorithm is used to denoise the image with the highest score above, so that the noise of the image is greatly reduced. A clearer image result is obtained, which more accurately determines the position and geometry of the unknown medium, and provides a reference value for imaging of the CT system.

Keywords: CT system, Filtered back-projection algorithm, Image reconstruction, Gray variance method, Image Denoising

1. INTRODUCTION

With the development of contemporary medicine and the improvement of people's living standards, many hospitals urgently need advanced CT systems to diagnose patients' conditions. How to use CT to reconstruct better images quickly is the key to better CT diagnosis. However, deviations often occur during the CT system's installation, which will affect the imaging quality. So it is necessary to perform parameter calibration on the installed CT system and image the samples of the unknown structure accordingly.

Modern CT systems are very complex, and different reconstruction algorithms take different amounts of time to simulate this complexity. Kim, Daehong ^[1] and others used the total variation denoising algorithm in the CT imaging system to verify the feasibility of sinogram reconstruction using a repair method based on a sinusoid with decomposition. Wang, Linyuan ^[2] and others studied the condition numbers and singularities of system matrices and regular matrices using singular value decomposition methods and proposed an empirical lower bound estimation method, which helps to estimate the number of projection views needed for accurate reconstruction. Do, Synho ^[3] and others systematically tested and examined the role of the high-fidelity system model using raw data in an iterative image reconstruction method that minimizes the energy function. The iterative image reconstruction algorithm improves the image successively through several iterations. It has the advantages of improving spatial resolution under high contrast and reducing noise at low contrast, but the disadvantage is that the operation speed is slow and is sensitive to the dose. Image quality deteriorates when the dose is low. Rowley, Lisa M ^[4] and others used the Bayesian penalty likelihood algorithm to optimize the reconstructed image. However, the key of the Bayesian algorithm is to find a suitable energy function to protect the edges of the image while denoising. But it is difficult to choose the right energy function.

The filtered back-projection reconstructed image is convoluted on the basis of back-projection, eliminating the edge sharpening effect ^[5] caused by pure back-projection, compensating the high-frequency components in the projection and reducing the density of the projection center. And ensure the edges are clear and the interior is evenly distributed. Image reconstruction with filtered back-projection is fast. Spatial and density resolution are relatively high. But in the filtered back-projection reconstruction algorithm, the design of the filter function is very crucial. Common filter functions are the Ramp-Lak filter, Shepp-Logan filter, and Lewitt filter. But the advantages and disadvantages of the three filters are different.

Based on these theories above, this paper uses the filtered back-projection algorithm (FBP algorithm) based on the Ramp-Lak, Shepp-Logan and Lewitt filter functions according to the data and questions in the National College Students Mathematical Modeling Contest A (2017) ^[6]. The image of the medium is reconstructed in three different ways to explore the difference. The gray-scale variance function is used to calculate the sharpness of the image. Then the clearest image is selected, and that image is denoised by the Wiener algorithm. This model provides a reference value for CT imaging.

2. MATERIALS AND METHODS

The basic idea of image reconstruction is to select the original density function, introduce the

filter function, solve the modified density function, and back-projection. Based on this idea, it is necessary to obtain the specific situation of the projected image first. Therefore, based on the received information of the unknown medium from the CT system, an image in which the X-ray illuminates the template from 180 directions is drawn. Since there are 512 detectors in each direction, a raw projection data distribution image of 512x180 is obtained, as shown in Figure 1.

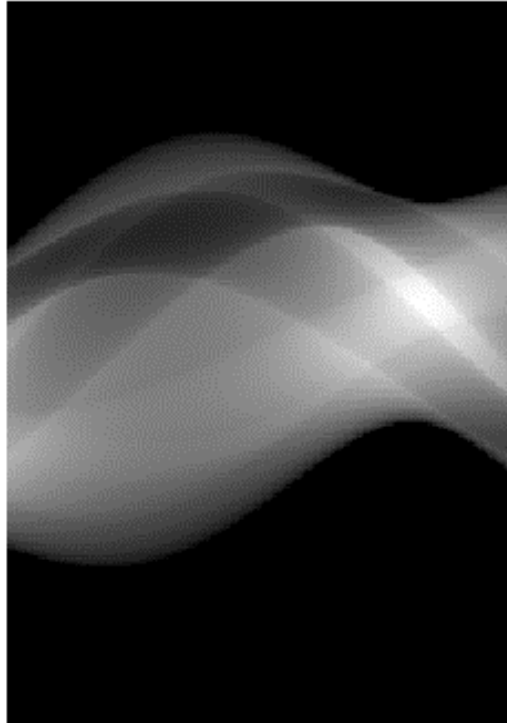


Figure 1: Raw projection data distribution image

However, this is not enough to obtain the clear projected image of the medium. In order to obtain the position and geometry accurately, the following three filter functions: Ramp-Lak, Shepp-Logan, and Lewitt filter functions will be introduced to correct the original projection data. Then a filtered back-projection model is established to reconstruct the image. The algorithm can be divided into the following five steps:

- Step1: Determine the sampling points and angular direction sampling points and their sampling data.
- Step2: Determine the Ramp-Lak, Shepp-Logan, and Lewitt filter back-projection function formulas.
- Step3: Calculate the discrete convolution.
- Step4: Beam calculation and linear interpolation.

Step5: Reconstruct the image of any point (x_i, y_j) using three filtered back-projection algorithms.

2.1 Determine the sampling points and angular direction sampling points

Set the number of translation sampling points to N_t . For the received data of 512 detectors, according to the parity rule, extract two sets of sampling points, namely uneven number sampling point group and even number sampling point group. And two sets of sampling points are respectively calculated. Where, $N_t=256$

$$\Delta x_r = d \quad (1)$$

$x_r = nd$, and x_r is the rotation coordinates.

Set the number of angular direction sampling points to N_ϕ . In this paper, $N_\phi=180$, angle increment $\Delta\phi = \Delta = \frac{\pi}{180}$, and we have equation (2).

$$\phi = m\Delta \quad (2)$$

It is easy to know that the picture pixels are 256×256 , as shown in Figure 2.

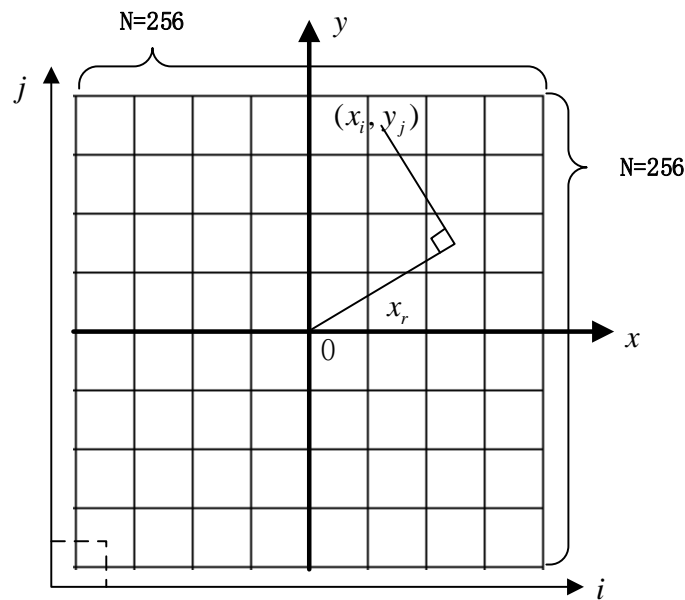


Figure 2: Pixel coordinates used for image reconstruction

In the figure above, the pixel position is recorded as (i, j) , i is the coordinate of the pixel in the x direction, j is the coordinate of the pixel in the y direction. The minimum of i and j is 1, so the pixel coordinate of the lower left corner in Figure 2 is (1, 1).

2.2 Determine the R-L, S-L and Lewitt filter back-projection function formulas.

In the filtered back-projection algorithm, commonly used filter functions include Ramp-Lak filter function, Shepp-Logan filter function and Lewitt filter function. In this paper, these three kinds of filter functions above will be used for back-projection imaging calculation. The formulas [7] are as follows. Due to space limitations, this paper will not deduct them.

The sampling sequence of the Ramp-Lak filter function is

$$h_{R-L}(nd) = \begin{cases} \frac{1}{4d^2}, n = 0 \\ 0, n = \text{even number} \\ -\frac{1}{n^2\pi^2d^2}, n = \text{uneven number} \end{cases} \quad (3)$$

The sampling sequence of the Shepp-Logan filter function is

$$h_{S-L}(nd) = \frac{-2}{\pi^2d^2(4n^2-1)}, n = 0, \pm 1, \pm 2, \dots \quad (4)$$

The sampling sequence of the Lewitt filter function is

$$h_{Lewitt}(nd) = \begin{cases} \frac{1-\frac{2}{3}esp}{4d^2}, n = 0 \\ -\frac{1-esp}{n^2\pi^2d^2}, n = \text{uneven number} \\ -\frac{esp}{t^2\pi^2d^2}, n = \text{even number} \end{cases} \quad (5)$$

2.3 Calculate the Discrete Convolution

When the rotation angle is ϕ_m , adopt projection $p(x_r, \phi_m)$, and determine the projection function $h(x_r)$. And the filtering projection is performed

$$\tilde{p}(x_r, \phi_m) = p(x_r, \phi_m) * h(x_r) \quad (6)$$

I.e.:

$$\bar{p}(x_r, \phi_m) = \int_{-\infty}^{\infty} p(x_r - x'_r)h(x'_r)dx'_r \quad (7)$$

Since the data acquisition is spatially discrete, and the amplitude is also discrete after A/D conversion, it should be discretely convolved.

From the above we know that $x_r = nd$, $\phi = m\Delta$. So correspondently x_r takes the variable n , and ϕ_m takes the variable m , the expression is

$$\tilde{p}(n, m) = p(n, m) * h(n) = \sum_{l=-N_l}^{N_l} p(n-l, m)h(l) \quad (8)$$

2.4 Beam Calculation and linear Interpolation

In CT image reconstruction, interpolation is a critical step. And interpolation is also the purpose of beam calculation. We have obtained the discrete filter function $h(n)$ and the projection data $p(n, m)$ from above. Consider them to be discrete convolution, we can obtain the filtered projection data $\tilde{p}(n, m)$, and then we perform linearly interpolate. We have

$$\tilde{p}(x_r, m) = \tilde{p}(n, m) * \psi(x_r)$$

From the above we know that both $x_r = nd$ and $\phi = m\Delta$ are discrete. For a certain point (x_i, y_j) in space, at a certain angle of view $\phi = \phi_m = m\Delta$, there is

$$x_{r,m} = x_i \cos \phi_m + y_j \sin \phi_m \quad (9)$$

Since (x_i, y_j) is the pixel coordinate of any point in space, $x_{r,m}$ obtained by the formula above is not exactly $\phi = m\Delta$ integer multiple of d , but may be located at between n_0d and $(n_0 + 1)d$, i.e.

$$x_{r,m} = (n_0 + \delta)d, 0 < \delta < 1 \quad (10)$$

because

$$\tilde{p}(x_{r,m}, \phi_m) = \tilde{p}_{m\Delta}(n_0d) + \frac{\tilde{p}_{m\Delta}[(n_0+1)d] - \tilde{p}_{m\Delta}(n_0d)}{d} (x_{r,m} - n_0d) \quad (11)$$

Ignore the subscript associated with the fixed viewing angle $m\Delta$ and let $d=1$, there is

$$\tilde{p}(n_0 + \delta) = (1 - \delta)\tilde{p}(n_0) + \delta\tilde{p}(n_0 + 1) \quad (12)$$

In order to get n_0 and δ in the interpolation formula, we use beam calculation to find a solution.

As mentioned before, the image area is usually divided into $N \times N$ pixels. When the ray rotates around the center of the image area after translation, the beam is translated from the first beam to the other end for each angle of view. The number of the bundle is also gradually increasing, as shown in Figure 3.

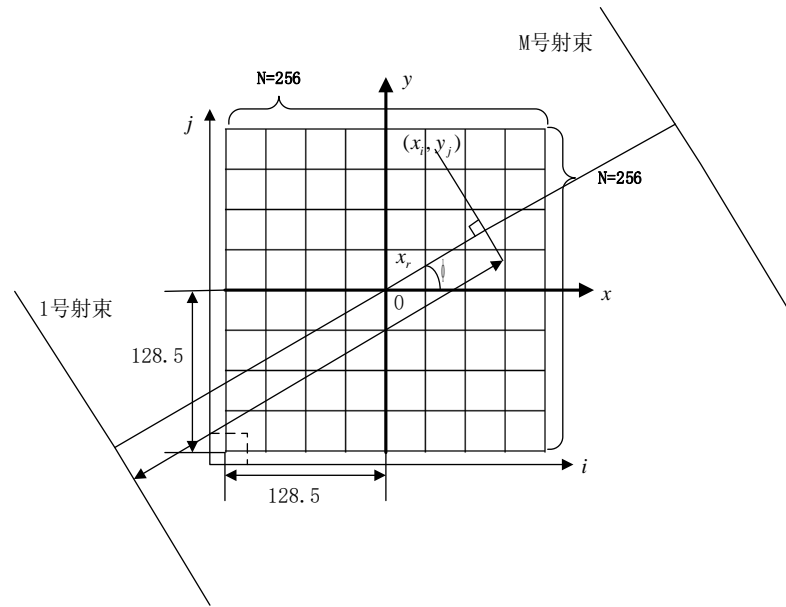


Figure 3: Beam bundle calculation

When one end of the beam is used as the origin, a negative value can be avoided in the actual calculation. So this coordinate is marked as \tilde{x}_r . Since the width of the pixel is 1, the minimum value of i and j is also 1, so for any pixel (x_i, y_j) and any viewing angle ϕ , there is

$$x_r = x_i \cos \phi + y_j \sin \phi = \left(i - \frac{N+1}{2}\right) \cos \phi + \left(j - \frac{N+1}{2}\right) \sin \phi \quad (13)$$

When x_r is converted to \tilde{x}_r , there is

$$\tilde{x}_r = x_r + \frac{M+1}{2} = \left(i - \frac{N+1}{2}\right) \cos \phi + \left(j - \frac{N+1}{2}\right) \sin \phi + \frac{M+1}{2} \quad (14)$$

I.e. :

$$\tilde{x}_r = \text{integer}(n_0) + \text{decimal}(\delta) \quad (15)$$

Where, M is the number of beams, n_0 is the beam number sought, and \tilde{x}_r corresponding to (x_i, y_j) is between the number n_0 beam and the number $(n_0 + 1)$ beam, and is δ apart from number n_0 beam.

2.5 Reconstructing Arbitrary Point Images Using Filtered Back-projection Algorithm

Combining the linear interpolation method above, the filtered projection value at \tilde{x}_r corresponding to any pixel point (x, y) is obtained according to a finite number of filtered projection values. And then any point is reconstructed by back projection using the above

formula:

$$\hat{a}(r, \theta) = \int_0^\pi g[r \cos(\theta - \phi), \phi] d\phi \quad (16)$$

$$a(x, y) = \hat{a}(r, \theta) = \int_0^\pi \tilde{p}(x_r, \phi) |x_r = x \cos \phi + y \sin \phi d\phi \quad (17)$$

When using programming to find a solution, replace (x, y) with (i, j) , x_r is represented by \tilde{x}_r , and let $\phi = m\Delta$. Δ is the angular increment, $m = 1, 2, \dots, N_\phi$ ($= 180$), when $\phi = m\Delta$, there is

$$\tilde{x}_r | \phi = m\Delta == \left(i - \frac{N+1}{2}\right) \cos(m\Delta) + \left(j - \frac{N+1}{2}\right) \sin(m\Delta) + \frac{M+1}{2} \quad (18)$$

And the expression (17) is

$$a(i, j) = \sum_{m=1}^{N_\phi} \tilde{p}[\tilde{x}_{r,m}(i, j), m\Delta] \quad (19)$$

I.e.:

$$a_m(i, j) = \sum_{m=1}^{N_\phi} \tilde{p}[\tilde{x}_{r,m}(i, j), m'\Delta] \quad (20)$$

notice that $a_0(i, j) = 0$, so the recursive formula of back projection reconstruction can be turned into

$$a_m(i, j) = a_{m-1}(i, j) + \tilde{p}[\tilde{x}_{r,m}(i, j), m\Delta] \quad (21)$$

In this equation, $m = 1, 2, \dots, 180$.

From algorithm above we can know that when the CT image is reconstructed by the filtered back projection, the first back projection is performed on all the pixels after the first filtering action. And then the corresponding pixels are correspondingly performed back-projection after the subsequent filtering effects until the last back-projection is done. Due to the fastness of the reconstruction, all pixels can be considered to be almost reconstructed together.

Then repeat algorithm steps by introducing Ramp-Lak, Shepp-Logan and Lewitt filter functions respectively to implement the computer simulation of the filtered back-projection algorithm. The results are shown in Figure 4, Figure 5 and Figure 6.

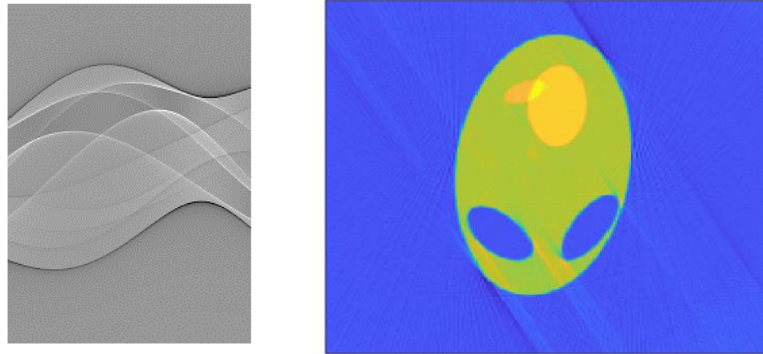


Figure 4: Projection data processed by Ramp-Lak filter and reconstructed image after projection

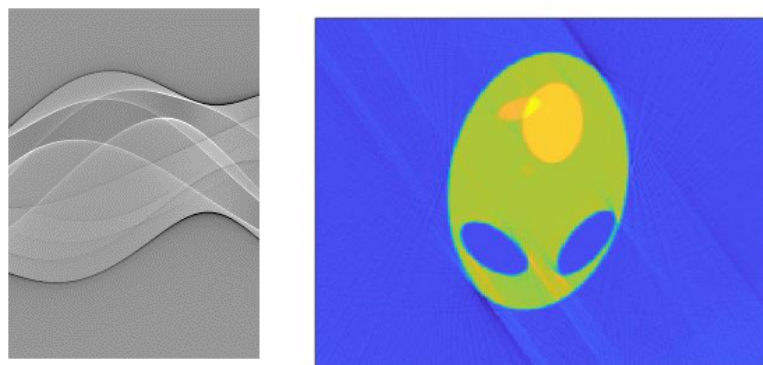


Figure 5: Projection data processed by Shepp-Logan filter and reconstructed image after projection

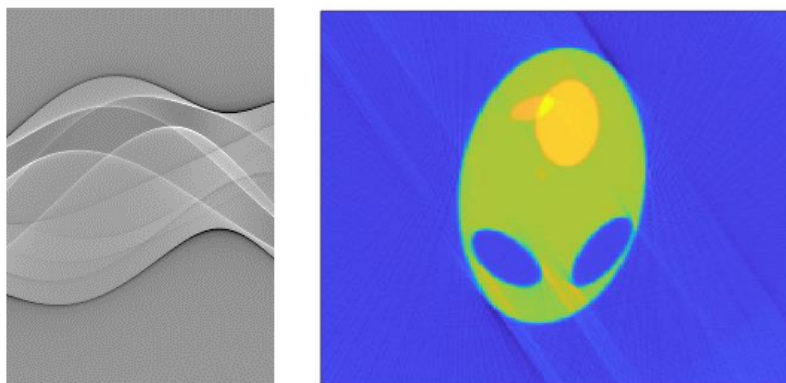


Figure 6: Projection data processed by Lewitt filter and reconstructed image after projection

It can be seen from Figure 4, Figure 5 and Figure 6 that the images reconstructed by the three filtered back-projection algorithms successfully express the features of the original object. The images are clearer and the reconstruction quality is better. However, the measurement of the quality of imaging and the advantages and disadvantages of the three methods are not known. In the following part we will introduce the gray-scale variance method to compare the clarity of the three images.

2.6 Calculate the Sharpness of Three Sets of Images Using the Gray-scale Variance Function

In the quality evaluation of non-reference images, the sharpness of the image is an important indicator to measure the quality of the image. It can better correspond to the subjective feelings of the human, and the sharpness of the image mainly represents the degree of blurring of the image. Therefore, the gray-scale variance function (SMD) will be introduced below to analyze and solve the sharpness of the three sets of images.

When fully focused, the image is the clearest. And the number of high-frequency components in the image will also reach the top. Therefore, the grayscale change can be used as the basis for the focus evaluation. The formula of the gray-scale variance method is as follows:

$$D(f) = \sum_y \sum_x (|f(x, y) - f(x, y - 1)| + |f(x, y) - f(x + 1, y)|) \quad (22)$$

The gray-scale variance evaluation function has better computational performance, but its disadvantages are also obvious. The sensitivity is not high near the focus and when the function is too flat near the extreme point, the focus accuracy is difficult to improve. Therefore, the gray-scale variance product method (SMD2) is used to solve the sharpness of the three sets of images, that is, multiplying two gray-scale differences in each pixel and then accumulating them pixel by pixel. The formula is

$$D(f') = \sum_y \sum_x |f(x, y) - f(x + 1, y)| * |f(x, y) - f(x, y + 1)| \quad (23)$$

The image scores of the three filtered back projection algorithms are calculated as follows:

Table 1: image scores of the three filtered back projection algorithms

Ramp-Lak's score	Shepp-Logan's score	Lewitt's score
394936	355261	334415

It can be seen from Table 1 that the image obtained by the filtered back projection algorithm using the Ramp-Lak filter function has the highest score, that is, the reconstructed image obtained by the back-projection algorithm using the Shepp-Logan filter function has the highest resolution. Therefore, the Ramp-Lak filter function can be considered as the most suitable function for the filtered back projection algorithm.

2.7 Noise Reduction of Images Using Wiener Algorithm

It is known from the above, the Ramp-Lak filter function can be considered as the most suitable function for the filtered back projection algorithm. However, this image inevitably contains more noise. Therefore, the commonly used noise reduction method, Wiener algorithm, is used to denoise the image obtained by the back-projection algorithm using the Ramp-Lak filter function.

The Wiener algorithm is based on the minimization criterion of mean-square error, which minimizes the error of the mean squared error between the final processed image $f'(x, y)$ and the original image $f(x, y)$, which significantly reduces image noise and improves image sharpness. Therefore, the Wiener algorithm is also called the optimal linear filter^[8]. The non-blind image restoration Wiener deconvolution time domain expression is:

$$f'(x, y) = TF^{-1}[AG] \tag{24}$$

Where, A is a linear Wiener filter whose expression in the frequency domain is

$$A = \frac{H^*(u,v)}{|H(u,v)|^2 + \frac{P_n(u,v)}{P_f(u,v)}} \tag{25}$$

Where, $H^*(u, v)$ is the conjugate function of the system degradation function $H(u, v)$, and $P_f(u, v)$ and $P_n(u, v)$ are the power spectra of the original image and the noise image, respectively. ^[9]

After Wiener filtering, the image obtained is shown in Figure 7:

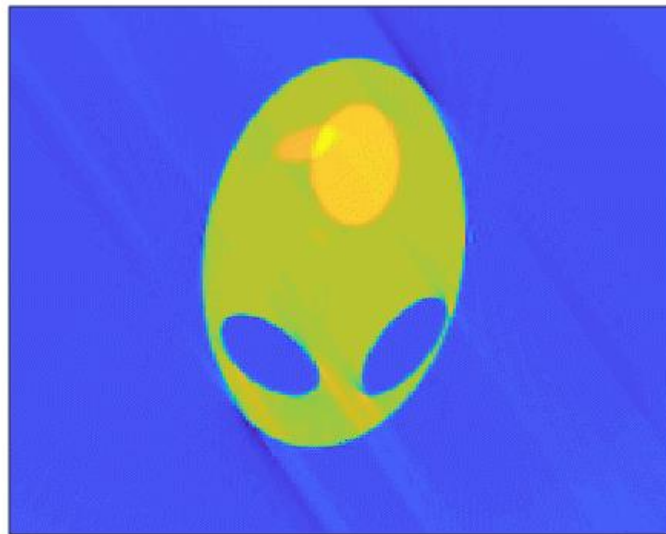


Figure 7: Denoised image using Wiener algorithm

After Wiener noise reduction processing, the optimized solution of the reconstructed image using the Ramp-Lak filter function is obtained. Figure 8 and Figure 9 are partial plots of the absorption rate gradation processing results obtained before and after noise reduction process.

-3.8328	-3.78204	-2.29264	-3.52684	-1.07706	0.967309	-2.93098	-1.71128	-0.08068	0.026522	0.741289	-4.38749	-0.34543	1.305295	0.919514	-2.25694	-1.6136	-1.10432	0.053875	1.894577	0.108149	-1.77923	0.137015
-3.89876	-7.53448	-3.01067	-0.89582	-1.80501	-2.04637	-2.80286	1.012092	-0.08942	-2.89167	-0.34644	-1.568	1.270329	-1.07526	-1.61614	-0.95618	0.266998	-0.60721	0.417117	-0.11062	-1.09822	-0.34279	1.737762
-5.19757	-6.21549	-2.31907	-2.4913	-4.37654	-0.18455	-0.34474	-1.15658	-2.21121	-1.63056	1.489818	-1.54617	-1.91729	-1.96294	1.140789	0.874251	-1.61284	-1.44345	-0.1189	0.812106	0.283853	-0.43479	-0.44611
-1.2023	-5.03014	-6.38769	-4.75603	-0.3053	-0.9433	-3.02377	-1.7034	0.159588	-0.7733	-2.46065	-1.07451	-0.64966	0.119211	0.469308	-1.10588	-2.13556	1.438999	0.132625	-0.40429	-1.72349	-0.72765	1.66871
1.89299	-6.65118	-5.41002	-3.65053	-1.87706	-4.44558	-0.19202	0.166811	-3.61285	-1.83072	-0.42569	1.04063	-1.45131	-2.38206	0.832376	0.474189	-0.71835	-1.45845	-1.58873	0.763079	-0.1974	1.572331	-0.04733
1.159409	-0.59375	-0.03296	-7.97261	-2.12204	-2.24776	-2.27291	-2.53797	-2.02021	1.402002	-2.59428	-0.40944	-1.1692	0.058081	0.277226	-2.40883	-0.83134	-0.32151	0.619833	0.050928	-2.29521	0.943169	1.136574
3.4975	-0.24978	-5.38199	-4.51528	-4.85892	-4.85421	-1.85232	0.498114	-2.66856	-1.73708	-0.82259	0.820319	-2.12813	-1.98056	-0.31453	0.176681	0.033463	-2.2487	-0.4283	-0.09128	0.257152	0.86429	-1.62674
0.372147	3.277476	-0.06549	-7.41635	-6.7495	-1.64261	-3.11887	-1.95968	-1.58503	-0.56706	-1.81055	-1.83227	0.291848	-1.18091	-1.2849	-1.06997	-0.09215	-0.70257	0.605349	-2.68428	-1.85001	0.680981	0.592016
3.520107	0.579984	1.004087	-3.35689	-6.68091	-4.99671	-2.49529	-2.21869	-2.38756	-1.52477	-0.33298	-1.48939	-1.71837	-1.30243	-0.222	-0.95071	-1.2085	-1.92346	-0.26311	-0.55571	0.890917	0.080242	-0.72644
1.62347	2.602119	3.239448	-1.08134	-5.34352	-7.43062	-4.03593	-3.11451	-0.87441	-2.8289	-1.07736	-1.30769	-2.15741	-1.90483	-1.57631	-0.25714	1.418749	-2.65613	-0.63971	-1.62831	1.238625	0.225393	-0.05006
2.486997	1.794831	3.395157	0.429711	-3.50828	-7.48932	-4.93906	-3.40131	-2.3554	-2.88961	-1.8675	-1.43823	-1.81209	-0.03142	-1.89309	0.212815	-0.16932	-2.68744	-0.33731	-2.52953	0.933938	-0.63138	-0.59782
1.696706	1.842457	3.416597	2.087376	-0.98923	-6.46192	-7.73771	-4.80573	-2.93284	-1.93477	-1.46374	-1.4355	-2.51475	-0.42767	-2.41734	-0.91011	-1.25486	-1.36626	-0.54348	-0.74491	1.193484	-1.37573	-0.27024
1.583735	1.873392	3.080333	0.945283	1.234641	-2.64712	-8.30904	-5.09504	-3.72467	-2.5456	-3.21059	-1.63046	-1.99464	-1.0149	-1.5873	-0.97881	-1.19874	0.023581	-1.70279	-0.54579	0.508994	-0.57207	-0.64455
2.047828	1.716132	2.922669	1.038391	2.989519	-1.02486	-6.83023	-7.76498	-4.58494	-3.05766	-2.85403	-0.89486	-1.53515	-2.05916	-0.89888	-1.96368	-0.38511	0.328457	-2.99757	0.703895	-0.6859	-0.29777	-0.13944
2.035928	1.956207	2.524682	1.05434	3.393952	1.024644	-2.94466	-8.06065	-5.45559	-4.89238	-3.02216	-1.90807	-1.89354	-2.68177	0.046588	-1.77985	-0.67518	-0.15992	-2.47042	0.772111	-1.29785	0.254128	-0.80048
3.548459	1.782319	3.256027	2.164735	2.896498	1.578728	1.83014	-7.07136	-8.79148	-4.36433	-2.81111	-2.46907	-1.49391	-2.98793	0.47643	-2.67541	-1.35221	0.345533	-2.69748	-0.022	-1.4192	-0.25058	-0.70866
0.748264	1.316427	2.649762	1.542606	1.15584	1.410931	3.27962	-4.87795	-9.4928	-6.71056	-5.7898	-3.47493	-2.21255	-3.32149	-0.46014	-4.06509	-1.59136	-0.2426	-3.24284	-1.36903	-1.44366	-0.26684	-0.85217
20.00147	6.243768	2.796138	2.261915	3.728117	2.13282	3.199474	-0.76687	-8.2501	-9.02608	-6.06765	-2.59032	-2.88233	-2.40173	-0.90102	-2.35905	-0.25372	-0.93968	-1.09394	-0.6054	-0.12654	0.546277	-0.74709
53.93622	47.85323	29.97591	9.943554	3.329124	1.361798	3.610805	0.947539	-2.40349	-9.79757	-7.66401	-4.20645	-3.62507	-2.40151	-2.61707	-1.72941	-1.6929	-1.90589	-0.28893	-2.63472	-0.84869	-0.2926	-0.79743
55.84631	56.38571	56.29215	50.8284	31.42037	7.575021	3.252005	1.070299	1.873952	-6.83533	-10.354	-6.22416	-4.54936	-2.08573	-3.02509	-1.74579	-1.9247	-1.3893	-0.4058	-2.47536	-0.04116	-0.63002	-0.20009
55.801	56.9124	55.47877	56.16102	57.32751	46.89419	22.22973	2.151605	3.516972	-2.46277	-10.88808	-8.4141	-5.01218	-4.12199	-2.72441	-0.613	-3.71356	-2.05033	0.135374	-2.53003	-0.49413	-0.68938	-0.01624
54.74611	57.79779	55.22635	57.42546	55.97743	55.81441	55.91198	35.25182	6.699026	-1.12649	-4.91839	-1.13378	-7.06257	-4.07662	-3.23176	-1.99161	-2.99288	-0.88248	-0.32734	-3.81104	-0.33363	-0.41702	-0.51375
55.73111	56.8343	55.28765	57.58593	55.95619	56.46726	56.23221	55.61536	43.92191	15.10553	-5.26446	-11.6257	-8.5852	-5.24979	-3.4346	-2.6475	-3.28	-2.05864	0.143684	-2.51933	-0.61484	-0.61215	-1.33956
55.29557	56.91993	56.16533	56.4589	55.97986	56.79505	56.11802	56.20909	56.02009	47.27451	18.5085	-10.3915	-13.8257	-7.22831	-4.60136	-0.97244	-3.64277	-1.78559	-0.78458	-3.33951	-0.81454	-0.49195	-1.1572
55.26757	57.46233	55.29471	57.48782	54.91232	57.58162	55.40301	57.02827	55.76608	56.87091	47.00765	16.03562	-10.3572	-13.6335	-6.84821	-2.88382	-2.77665	-2.89331	-0.81361	-0.59496	-1.94548	-0.86034	-1.26391
55.82863	57.1457	55.66527	56.57763	55.62209	57.41504	55.06821	57.37676	55.56872	56.84759	56.54508	46.38455	8.326694	-12.5416	-9.55378	-5.29907	-4.18149	-2.13403	-2.34445	-1.07852	-1.11271	-0.38566	-0.46313
56.46313	56.39853	55.70258	56.86028	55.65989	57.39796	55.18744	57.17884	55.91504	56.83455	55.76446	57.29695	42.0416	-3.08933	-1.10595	-4.06406	-6.38065	-2.71908	-2.49248	-1.88591	-2.16558	-0.93122	0.203883
56.23755	56.4017	55.27849	56.9617	55.26601	57.57449	55.57852	57.02448	55.9736	56.26081	56.75048	55.76377	56.43516	32.77001	-10.6221	-8.00166	-2.71091	-3.52262	-3.73308	-1.75567	-3.44727	-1.21417	-0.83684
55.70405	56.2582	55.70643	56.80784	55.50523	57.28296	55.6211	56.63691	56.13774	56.26741	56.76835	55.90263	56.79352	52.55552	17.31162	-11.5774	-6.84146	-0.9647	-3.7758	-1.40129	-3.78202	-1.45392	-1.35968
54.96467	56.57625	55.91411	56.44564	55.61459	56.87257	55.97322	55.98515	56.6303	56.23366	56.75301	55.78205	56.91015	55.72363	43.70419	4.090676	-9.19597	-6.8475	-1.62833	-0.58796	-2.42046	-3.04932	-1.09284

Figure 8: Partial data after Ramp-Lak filtered back-projection algorithm denoise

-4.52146	-3.66306	-2.92278	-2.00562	-1.57194	-1.43971	-0.89738	-1.11735	-0.90269	-0.4159	-1.05332	-0.64174	-1.06815	-0.05895	-0.44445	-0.53875	-0.89613	-0.69339	-0.2916	0.03015	0.038505	0.041749	-0.16015
-4.21397	-4.2315	-3.56315	-2.42166	-1.71513	-1.62231	-1.022	-1.14618	-0.97031	-0.55471	-1.12363	-0.73438	-1.13633	-0.25346	-0.40307	-0.53935	-0.93925	-0.64026	-0.02298	0.249104	-0.07411	-0.20382	-0.45051
-3.51704	-4.53291	-4.29341	-2.92749	-1.97825	-1.75916	-1.24372	-1.12892	-1.03161	-0.97265	-1.20017	-0.75584	-0.93381	-0.46907	-0.45698	-0.51947	-0.58676	-0.40691	0.01293	-0.20109	-0.41621	-0.1203	-0.18438
-2.25312	-4.05783	-4.76794	-3.50817	-2.55891	-1.74365	-1.31413	-1.32424	-1.39914	-1.25506	-0.80124	-0.7772	-1.09157	-0.64462	-0.1712	-0.19797	-0.6319	-0.83385	-0.20745	-0.22679	-0.00625	-0.00576	-0.48175
-0.70959	-3.02841	-5.05388	-4.16825	-3.14669	-1.93664	-1.9111	-1.67075	-1.19445	-1.35068	-0.79127	-1.02107	-0.65708	-0.43289	-0.40737	-0.57187	-0.78519	-0.54028	-0.08528	-0.51585	-0.22428	-0.07448	-0.1304
0.581392	-1.86331	-4.38423	-4.53571	-4.06044	-2.74698	-1.97087	-1.61021	-1.37116	-1.5	-0.50582	-0.7928	-0.84413	-0.91757	-0.58527	-0.27546	-0.81143	-0.77134	-0.52257	-0.32333	0.207451	-0.04369	-0.5245
1.473803	0.728227	-1.79158	-4.22458	-5.00793	-4.13882	-2.51558	-1.97643	-1.57226	-1.49291	-1.03293	-1.00246	-1.16888	-0.93368	-0.85863	-0.6127	-0.82944	-0.37399	-0.57784	-0.6462	-0.45825	-0.25531	-0.19147
1.934493	1.794816	-0.13522	-2.93894	-4.96649	-4.72155	-3.44588	-2.42111	-1.89562	-1.44318	-1.419	-1.27046	-1.40016	-1.22837	-1.08324	-0.58255	-0.82687	-0.60684	-1.16088	-0.54292	-0.40024	0.120185	-0.31552
1.965525	2.249578	0.956344	-1.32251	-4.38421	-5.21329	-4.45794	-2.86913	-2.39946	-1.79317	-1.6396	-1.46678	-1.46243	-1.40199	-0.88057	-0.51617	-0.91346	-0.94069	-1.46897	-0.32113	-0.21953	0.151491	-0.69584
2.02187	2.455309	1.969595	0.182879	-3.30968	-5.32618	-5.49068	-3.79966	-2.79305	-2.02495	-1.80481	-1.67492	-1.44773	-1.63721	-1.02279	-0.76073	-0.85219	-0.91509	-1.45923	-0.33969	-0.36871	0.074024	-0.80234
2.077819	2.352245	2.096126	1.121287	-1.8221	-4.53856	-5.65403	-4.8112	-3.29833	-2.54719	-2.04622	-1.92972	-1.36663	-1.52147	-1.00531	-1.13297	-0.92546	-1.02629	-1.15933	-0.4186	-0.41811	-0.16171	-0.86148
1.988383	2.242205	2.102515	1.858398	-0.31421	-3.25407	-5.63074	-5.75391	-4.04958	-2.9232	-2.11413	-1.94819	-1.50079	-1.60553	-1.36198	-1.28831	-0.85617	-1.01075	-0.76054	-0.5349	-0.20175	-0.25369	-0.77353
1.863077	2.193434	1.90127	2.131534	0.891865	-1.42524	-4.62619	-5.86351	-5.02017	-3.70529	-2.66842	-2.10483	-1.73473	-1.51319	-1.43531	-1.04677	-0.75436	-1.02641	-0.67205	-0.85726	-0.12892	-0.40833	-0.53266
2.24117	2.421139	2.046167	2.471201	1.681549	0.325547	-3.21459	-5.1951	-6.00482	-4.42596	-2.9193	-2.09799	-1.9915	-1.44748	-1.61374	-1.02303	-0.92415	-1.11821	-0.6886	-1.12382	-0.24924	-0.59397	-0.39701
1.867851	2.202008	2.027456	2.29316	1.804475	1.515766	-1.09703	-5.20379	-6.63523	-5.70336	-3.93805	-2.78613	-2.4937	-1.61426	-1.93874	-1.3418	-1.35512	-1.34294	-1.00963	-1.4656	-0.56033	-0.75392	-0.51588
1.57489	2.834416	2.668189	2.494632	2.096916	2.356914	0.079504	-3.84321	-6.32516	-6.54209	-4.81154	-3.31019	-2.64825	-1.7983	-2.07727	-1.46462	-1.45929	-1.22981	-1.09638	-1.33557	-0.55078	-0.58538	-0.47421
20.13154	6.683116	3.320753	3.029632	2.985195	2.578731	1.144242	-1.21124	-5.32809	-6.97522	-6.1586	-4.29035	-3.02404	-2.31366	-2.25072	-1.74109	-1.64219	-1.25021	-1.36923	-1.29486	-0.78236	-0.53653	-0.44073
53.43854	47.49231	29.95388	10.32481	3.823876	2.295828	2.486988	0.550968	-3.18455	-6.23324	-6.98506	-5.36259	-3.45185	-2.72099	-2.14071	-1.80542	-1.54894	-1.09943	-1.30434	-0.94673	-0.7898	-0.34859	-0.3428
55.11564	55.4829	55.71079	50.40996	31.36519	8.003779	3.69591	2.008566	-0.64512	-5.82152	-7.43768	-6.78112	-4.52673	-3.35138	-2.34044	-2.19844	-1.86276	-1.47067	-1.505	-1.06483	-1.18179	-0.44553	-0.51223
56.4527	56.05407	55.83423	55.40857	56.72722	46.57927	22.32693	2.679999	3.618041	-2.57272	-6.95042	-7.63925	-5.87606	-3.98775	-2.624	-2.44031	-1.92263	-1.50567	-1.52626	-1.14257	-1.26908	-0.3706	-0.55875
56.63093	56.9795	56.5233	56.2659	55.51216	55.27688	55.36756	35.13468	7.06759	-0.77098	-4.83337	-8.12234	-7.27621	-4.83324	-3.12125	-2.73659	-2.24778	-1.66957	-1.54446	-1.15014	-1.33573	-0.55897	-0.61038
56.60554	56.0046	56.63351	56.22923	56.49561	56.13916	55.44639	55.12504	43.64209	15.23117	-4.88219	-10.684	-8.82034	-6.36621	-3.71489	-2.97721	-2.25043	-1.73451	-1.7072	-1.3779	-1.43933	-0.6994	-0.52361
56.58386	56.02872	56.61077	56.12541	56.58055	56.18061	56.38332	54.76416	55.45015	46.95381	18.52653	-9.9145	-12.5729	-8.19598	-5.27773	-3.45415	-2.54897	-1.98794	-1.62732	-1.25369	-1.31034	-1.02222	-0.62002
56.55915	56.11612	56.57529	56.01822	56.5367	56.09947	56.55501	56.06203	55.44023	56.1753	46.73136	16.06916	-9.94432	-11.6558	-7.03813	-4.52884	-2.95213	-2.37294	-1.75206	-1.42537	-1.18041	-0.95499	-0.49621
56.45746	56.13649	56.51054	55.97584	56.61274	56.02751	56.62635	56.05471	56.59853	55.23556	55.96638	46.0979	8.455131	-12.0048	-7.80916	-5.89414	-3.70357	-2.97064	-1.88404	-1.60374	-1.21782	-1.00268	-0.48261
56.26471	56.12462	56.33243	55.95488	56.59279	56.08552	56.64464	56.09684	56.55337	56.27337	55.38314	56.74095	41.81389	-2.78764	-10.4463	-6.8748	-4.33484	-3.35764	-2.40732	-2.22396	-1.55297	-1.1503	-0.4872
56.05742	56.01674	56.26397	55.97205	56.59071	56.11929	56.60919	56.13929	56.46993	56.29894	56.40105	55.13144	55.98572	32.63632	-10.127	-7.37297	-5.19806	-3.68231	-2.47229	-2.71546	-2.00412	-1.6652	-0.75623
55.8966	55.89349	56.26115	55.94445	56.48123	56.14319	56.50549	56.17345	56.35001	56.41949	56.2758	56.42879	55.84208	52.19724	17.35868	-10.9361	-5.60815	-4.35782	-2.69077	-2.50354	-2.12356	-2.07295	-1.14223
55.688	55.77041	56.25239	55.97593	56.34484	56.1747	56.3159	56.20771	56.24498	56.51887	56.21696	56.54374	55.80888	55.18225	43.44514	4.334688	-8.3662	-4.90574	-3.24904	-2.34499	-1.84931	-1.83948	-1.31092

Figure 9: Partial data after Wiener filtered back-projection algorithm denoise

It can be concluded from Fig. 8 and Fig. 9 that after noise reduction processing, the number of noises of the image is greatly reduced, the contour of the image is clearer, and the noise reduction effect is better.

3. CONCLUSION

In this paper, based on the back-projection image in the CT system, three image reconstruction models based on Ramp-Lak, Shepp-Logan and Lewitt filter functions are established. The reconstructed image quality obtained by filtered back-projection algorithm is explored and determined. The location and geometry of an unknown medium in a square tray. Firstly, based on the established filtered back-projection algorithm, the image is back-projected and reconstructed by Ramp-Lak filter function to obtain the position and geometry of the medium. However, considering the limitation of the back-projection of a single filter function, the Shepp-Logan and Lewitt filter functions are used to repeat the above algorithm steps, and three sets of reconstructed images are obtained. In order to further compare the advantages and disadvantages

of the back-projection image reconstruction using each filter function, the gray scale variance function is used to calculate the sharpness of the three sets of back-projection images, and the image quality obtained by using the Ramp-Lak filter function is the highest. Then the Wiener algorithm is used to denoise the selected image, and the reconstructed image with significantly reduced noise is obtained, which more accurately reflects the position and geometry of the unknown medium in the square tray, providing image reconstruction in the CT system. Reference.

The algorithm has the following disadvantages: Firstly, the reconstruction of the image relies too much on the experimental data. When the experimental data is unreliable, the image obtained by the reconstruction algorithm does not have high accuracy, thus lacks strict scientific. Secondly, there are many kinds of filter functions. In this paper, only three common filter functions are used for the reconstruction of back-projection images, and only the Wiener algorithm is used for noise reduction. It is not always possible to get the best back-projection reconstructed image. Accurately reflect the position and geometry of the unknown media in the tray.

However, the advantages of this model are also obvious: First, the development of modern science and technology provides a strong guarantee for the precision of CT, so that the data collected by the CT system has high accuracy, which can better ensure the back-projection reconstruction image. The accuracy. Secondly, the process of filtering the back-projection algorithm is simple and effective, and the position and geometry of the unknown medium in the tray can be obtained conveniently and quickly.

Therefore, the filtered back-projection image reconstruction model established in this paper can easily obtain the reconstructed image of the unknown medium when it guarantees certain reliability, which provides a simple and effective reference for future back-projection imaging using CT system parameters and related data.

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