

## **HIDDEN PATTERNS: THE MAGIC OF THE FIBONACCI NUMBERS**

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### **ABSTRACT**

The Fibonacci sequence is a sequence of numbers discovered by Italian mathematician Leonardo Fibonacci in the 13th century, these numbers are seen in his book "Liber Abaci." The sequence is a series of numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34... The next number in the series is found by adding the two terms before. Fibonacci numbers can be applied to the growth of every living organism, the splitting of a single cell, to the reproduction in a hive of bees to the pattern of reproduction of mankind. This research paper compiles the multiple examples of the Fibonacci numbers seen in nature from the leaves of a plant, branches of a tree, to the petals of a flower. In the paper is explained the link between the Fibonacci numbers and the human body, and the link between the Fibonacci series and the Golden ratio.

**Keywords:** Fibonacci, numbers, series, sequence, pattern

- The Fibonacci sequence is a sequence of numbers discovered by Italian mathematician Leonardo Fibonacci in the 13<sup>th</sup> century, these numbers are seen in his book "Liber Abaci." These numbers were discovered close to 800 years ago, I am compiling this information from multiple sources so that it is simplified and compressed in one article. The Fibonacci numbers are a sequence of numbers in which each term is the sum of the two preceding numbers. Or simply put, add the last two to get the next. The numbers are:

0      1      1      2      3      5      8      13      21      34      55      89      144  
         233    377    610    987

### Patterns in the Fibonacci numbers:

There are patterns that have been discovered in the Fibonacci numbers itself. To point out a few:

1. Look at the final digit in each Fibonacci number - the **units digit**: **0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...**  
Do you think there is a pattern in the final digits? **0, 1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0, 7, ...**

Yes, there is a pattern. With the help of long calculations a cyclic pattern is seen every 60 numbers.

2. If we look at the final **two digits** of the Fibonacci numbers. Is there a pattern here?

**0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...**

Yes, there is a pattern here too. After every 300 numbers the last two digits are repeated in the same sequence.

3. The third number in the series is **2**. It is seen that every **3rd** number is a multiple of **2** - (2, 8, 34, 144, 610, ...)

- The fourth number in the series is **3**. Every **4th** number is a multiple of **3**

- (3, 21, 144, ...)

- The fifth number in the series is **5**. Every **5th** number is a multiple of **5**

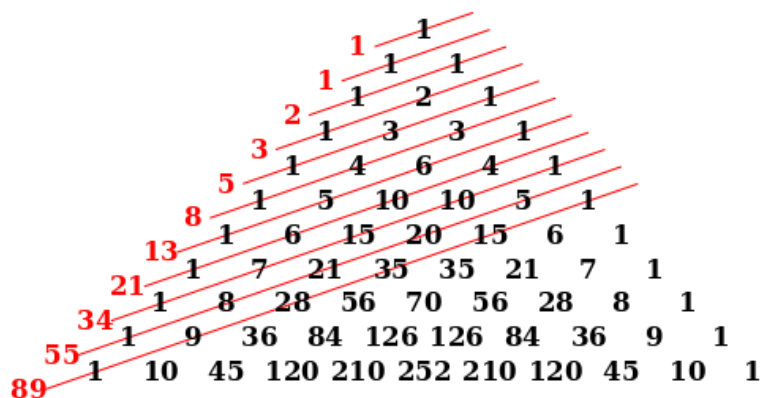
- (5, 55, 610, ...)

And so on (every **n**th number is a multiple of **x<sub>n</sub>**).<sup>1</sup>

- Fibonacci numbers can be applied to the growth of every living thing, the splitting of a single cell, to the reproduction in a hive of bees to the pattern of reproduction of mankind. Here are some examples of where they are seen in nature.

### Pascals triangle

The upper diagonals of pascals triangle sum to Fibonacci numbers:



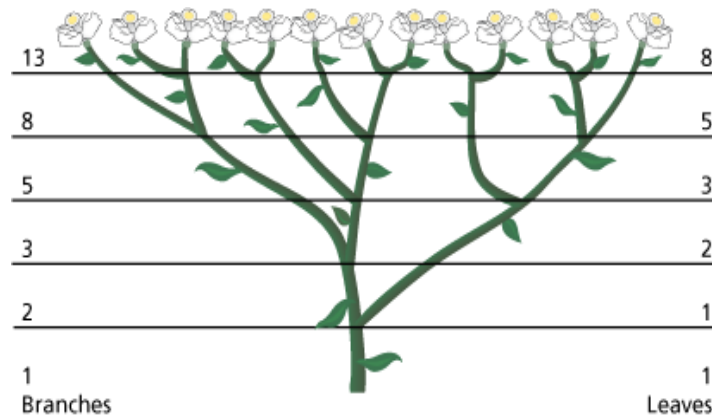
<sup>1</sup> Taken from <https://www.mathsisfun.com/numbers/fibonacci-sequence.html>

Nature

Some plants such as *Achillea ptarmica* branch in such a way that they always have a Fibonacci number of growing points. As we can see that the branches at every successive stage are showing Fibonacci numbers.

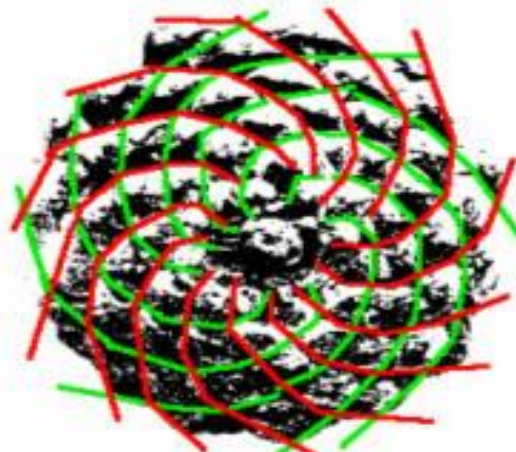
1. Leaves:

Leaves follow Fibonacci both when growing off branches and stems, and even in their veins. Like a tree, leaf veins branch off more and more outward in accordance with the Fibonacci Sequence.



2. Pinecones:

In the case of tapered pinecones or in pineapples, we see a double set of spirals – one going in a clockwise direction and one in the opposite direction. When these spirals are counted, the two sets are found to be consecutive Fibonacci number

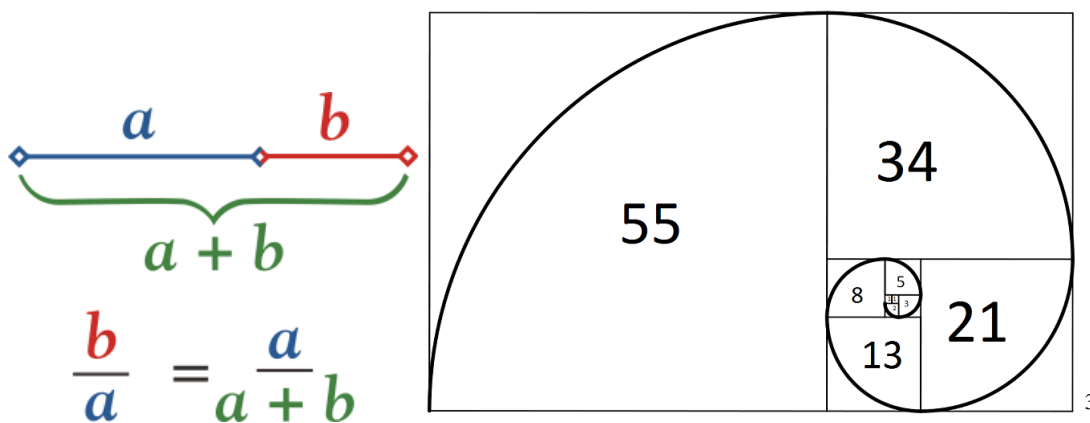


3. Fibonacci Petals:

3 petals	lily, iris
5 petals	buttercup, wild rose, larkspur, columbine
8 petals	delphiniums
13 petals	ragwort, corn marigold, cineraria
21 petals	aster, black-eyed susan, chicory
34 petals	plantain, pyethrum <sup>2</sup>

• The golden ratio and Fibonacci numbers:

The greeks were very interested in patellar numbers that they found intrinsically beautiful. One such number was the "golden ratio."The golden ratio is:



- Interestingly the ratio of successive Fibonacci numbers approaches the golden ratio as n approaches infinity. Phi is the number of the golden ratio which is equal to 1.618. The golden ratio is seen in nature as the number of female bees always outnumber the male bees in the ratio of 1.618 to 1.

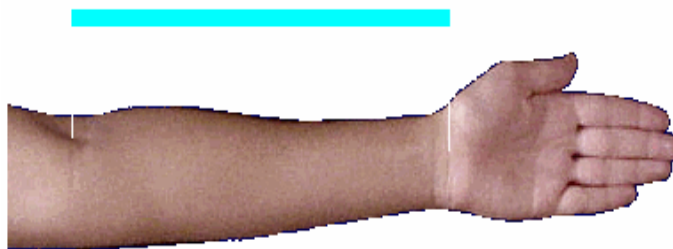
<sup>2</sup> Data taken from <https://www.goldennumber.net/plants/>

<sup>3</sup> <https://www.bigtrands.com/education/fibonacci-numbers-indicators-in-technical-analysis-charting/>

Sequence of terms	Ratios of Successive Terms	
$f_n$	$f_{n+1}/f_n$	
1		
1	1	
2	2	
3	1.5	
5	1.667	
8	1.6	
13	1.625	
21	1.6154	
34	1.619	
55	1.6176	
89	1.61818	
144	1.61797	4

Human body

- The distance between your shoulder and fingertips divided by the distance from your elbow to your fingertips is phi.
- We have 8 fingers in total( plus two thumbs), 5 digits on each hand, 3 bones in each finger, 2 bones in 1 thumb, and 1 thumb on each hand. Does this sound familiar?



- Below is a little program I have written which generates all the Fibonacci numbers till 1,000.

<http://cpp.sh/4vbsi>

- Fun fact:

Fibonacci Day is celebrated on November 23rd, it has the digits 11/23 which are a part of the sequence.

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<sup>4</sup> Image <https://math.temple.edu/~reich/Fib/fibo.html>

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Also taken from Columbia University notes.