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IMPROVING MATHEMATICS LEARNING BY ADDING CONCEPTUAL TO PROCEDURAL INSTRUCTION

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ABSTRACT

Today, the importance and growth of science, technology, engineering and math (STEM) fields have brought about an increase in demand for labor in which a proficiency in math is required. As students from elementary school to college look to develop their mathematical capabilities, different teaching formats of math courses have raised the question of which ones result in the greater mastery of the taught subject. As these courses provide the foundation from which many build their careers, the answer to this question is of the utmost importance. In the present experiment, we set out to compare whether adding conceptual instruction to procedural instruction improves learning compared to procedural instruction alone when students were taught a difficult math concept. 27 students were taught conditional probability in either a procedural only or procedural plus conceptual manner. Results showed that adding a modest amount of conceptual instruction to procedural instruction tripled problem-solving performance on a post-test. Results suggest that teaching procedures alone may not be sufficient as subject matter grows more complex and that adding conceptual instruction to procedural instruction may be necessary for subject matter mastery.

Introduction

Through the late 20th century, instructional learning consisted of two major frameworks: procedural learning and conceptual learning. We define conceptual knowledge as explicit or implicit understanding of the principles that govern a domain as well as the relationships between pieces of knowledge in a subject; we define procedural knowledge as understanding of the steps for solving problems. Conceptual understanding supports understanding mathematical procedure as a byproduct of connecting old information with new information. Procedural

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understanding is built on knowing the routine of a mathematical concept. It is also built on determining when to apply various concepts.

If students can remember rules and procedures, that does not show that they know enough about the topic. However, if a student is able to formulate why these rules and procedures exist and why they work, this shows a far more capable understanding on the topic. There has been a long history of debate among researchers in mathematical education on the relationship between conceptual and procedural knowledge. So far, researchers have proclaimed four theoretical perspectives on this matter. The first assumes that students gain conceptual knowledge at first and gain procedural knowledge through the constant application of said procedural knowledge; this is known as the concept-first perspective. Opposing this theory is the procedure-first perspective, which proposes that students first learn procedural knowledge and later use it as a foundation to build conceptual knowledge. Separate from both of the previous notions is the idea that procedural and conceptual knowledge develop independently, with neither influencing the other. The fourth perspective, the iterative perspective, asserts that the relationship between procedural and conceptual knowledge is bidirectional [9].

From these debates arose another question: does procedural or conceptual knowledge result in students' better ability to understand taught materials? With math being a field that plays a crucial role in human life and whose concepts range in complexity, it is necessary to research which teaching format results in a better understanding of mathematics. Previous studies have already investigated the posed question.

For example, Stump et al. researched adults' knowledge of concepts relating to slope, comparing their conceptual knowledge and procedural knowledge [8]. The results revealed that those with minimal conceptual knowledge had a difficult time differentiating between linear and other equations in addition to being unable to answer questions about rate of change. A similar study done by Faulkenberry et al. found that adults who did not have a strong grasp of the concepts of slope tended to rely on procedural knowledge [2]. On the other hand, those who did have a strong understanding of slope had knowledge on both concepts and procedures.

Another study done by Rittle-Johnson et al. examined the relationship between elementary school students' procedural and conceptual knowledge of mathematical equivalence and their ability to solve equivalence problems after a brief lesson; the study workedoff of the notion that conceptual and procedural understanding influence each other [7]. Their results revealed that children who were taught conceptually were better able to solve problems and develop procedural knowledge than the reverse.

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Relatedly, a study conducted by Ghazali & Zakaria sought to understand the relationship between students' procedural and conceptual knowledge related to algebraic concepts and their ability to algebraic problems that catered to procedures or concepts [3]. The study suggested that students should be taught procedurally and conceptually but emphasized the importance of conceptual knowledge, stating that it enables students to solve problems of various forms, including those that they had never come across before.

Given that empirical research suggests that both procedural and conceptual knowledge is important for mastering a topic, the question arises as to what to teach and when to teach it? Mahajan et al. conducted an experiment that explored the effectiveness of procedural vs. conceptual instruction for beginning and advanced computer programming concepts [5]. Their experiment employed a 2 x 2 design in which beginners with no computer programming experience learned to program loops in Python and experienced programmers learned lambda functions by either a procedural or conceptual method. Mahajan et al. found that beginning students performed equally well, regardless of teaching methods, while experienced programmers learned better when taught with a procedural method. Their results suggest that for beginning programming concepts, it matters little how students are taught since the concepts are simple to master. On the other hand, for advanced topics, it is important to have a procedural foundation before conceptual instruction can be maximally effective.

This hypothesis matches the knowledge acquisition framework developed by Leddo et al. [4]. Working with people at different skill levels, Leddo et al. noted a knowledge progression that went from factual (semantic) to procedural to conceptual as people became more and more proficient in their professions. This finding extends the landmark ACT-R framework, created by John Anderson [1] that argues that people progress from factual (declarative) to procedural knowledge.

The present research is designed to expand the work of Mahajan et al. by investigating whether, for advanced topics, adding conceptual instruction after procedural instruction is given can significantly boost problem solving performance. If so, this would lend support to Leddo et al.'s extension of Anderson's ACT-R framework.

In the present study, we focus on how adding conceptual instruction after procedural instruction affects the problem solving performance of math students. Procedural teaching refers to the understanding of definitions as well as the different methods applicable in solving problems, relying on memorization and basic structural knowledge. On the other hand, conceptual teaching refers to the understanding of how and why different methods work in solving said problems, taking the material to another level by allowing students to make more detailed observations and

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identify more complex relations using what they have been taught. The topic chosen was conditional probabilities, a topic that most students find difficult to learn. Our hypothesis is that adding a modest amount of conceptual instruction after giving students procedural instruction will greatly improve student performance.

Method

Participants

Participants in this study were 27 students, 9 of whom were middle schoolers and 18 of whom were high schoolers, located in Washington, DC, Maryland, and Virginia in the United States. Participants came from a variety of ethnic and socio-economic backgrounds and included both boys and girls. Some were enrolled in the public educational system while others were enrolled in private school. The classes they took were advertised as free math classes for students enthusiastic in learning about advanced mathematics. Participants were not told beforehand about the subjects to be taught in the classes so as to avoid their research about said subjects. The prerequisite for all participants was simply a basic knowledge of what probability is. Thus, no students who applied for the class ended up being rejected. All participants were taught as part of a group. No student who signed up missed their class.

Materials

There were two different Google Presentations made to represent the different formats of teaching: procedural and conceptual. All students were taught the principles of conditional probability and were asked to solve a practice problem as well as three post-test problems. However, the content of the presentation, depending on whether it was procedural or conceptual, was different. The procedural presentation mainly focused on introducing to students different methods that could be used to solve conditional probability problems such as formulas, diagrams, and tables. Learning said methods could help students calculate the correct answer to questions but did not give them a complete understanding of the logic behind them. Meanwhile, the conceptual presentation included methods, real-world applications, and additional information on the logic behind the application of different techniques as well as what differentiates conditional probability from other forms of probability. This additional information helped enable students to fully understand the logic and reasoning needed to adapt and efficiently solve conditional probability problems.

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Procedure

Each student was taught in a single online session through Google Meet or Zoom and was not given the ability to choose the teaching format of their liking. Instead, they were assigned a teaching method based on their time slot and the size of the group they were to take the lesson with. A total of 14 students were assigned to the Procedural condition and 13 students were assigned to the Conceptual condition. Procedural sessions averaged around 40 minutes each, whereas conceptual sessions averaged around 50 minutes. Students were encouraged to ask any question they had during any part of the lesson.

Both procedural and conceptual sessions started with a brief overview of the plan of the class. Students were then given a brief pretest. The pretest consisted of one question, designed to confirm whether or not participants had previously studied and mastered conditional probability. Students were given 5 minutes to complete the pretest, after which the procedural and conceptual classes' lesson plans diverged. In the procedural teaching, about 60% of the time was dedicated to teaching students about the definition of conditional probability, the formula for conditional probability, a walkthrough of solving a practice problem, and contingency tables. The other 40% of the class was dedicated to a post-test. On the other hand, in the conceptual teaching, 70% of the session was dedicated to teaching students about the definition of conditional probability, the formula for conditional probability, a walkthrough of solving a practice problem, an analysis of the differentiating aspects of conditional probability when compared to other forms of probability, probability trees, and real-life applications of conditional probability in today's world. The remaining 30% of the class was dedicated to the post-test. This was the same post-test that the procedural group was given. The post-test was designed to test students on their general understanding of conditional probability. Students were asked to solve three questions in 15 minutes. The three questions are listed numerically below.

1) The probability that it is Friday and that a student is absent is 0.04. The school week spans from Monday to Friday. What is the probability that a student is absent given that the day is Friday?

2) Lindsay took two biology exams. The probability of her getting above a B on both tests is 0.75. The probability of her getting above a B on the first test was 0.8. What is the probability of her getting above a B on the second test given that she has gotten an A on the first test?

3) You toss a coin with heads on one end and tails on the other three times. What is the probability that the coin lands on heads at least two times given that it has landed on heads at least once?

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Results and Discussion

The total number of the three post-test questions correctly answered was tabulated for each student. Each correct answer to a question was given a score of 1 and each incorrect answer was given a score of 0. The mean number of correct answers given was 2.62 for students in the Conceptual condition and .86 in the Procedural condition. This difference was statistically significant, $t = 5.01$, $df = 25$, $p < .0001$. Perhaps even more telling is an examination of the individual scores. While 8 of 14 participants in the Procedural condition got 0 questions correct in the post-test, 8 of 13 participants in the Conceptual condition got all three questions correct.

The difference in performance between the two conditions was dramatic. While it is true that participants in the Conceptual condition received an additional 10 minutes of session time, which, based on the Procedure section, translated to an average of 11 minutes more teaching time and roughly equal post-test time, this difference seems unlikely to account for the magnitude of the effect observed between conditions. Students in the Conceptual condition spent the same amount of time on the post-test as those in the Procedural condition, so any difference in performance could not be attributable to more time spent on problem solving. It is true that students in the Procedural condition averaged roughly 24 minutes of instruction time compared to 35 minutes of instruction time in the Conceptual condition, an increase of nearly 50% across conditions. However, this extra time of instruction seemed to offer a much greater return on investment per minute than did the procedural instruction alone, given that performance rose 200% for just the extra 11 minutes of instruction, going from a situation where most students got 0% correct on the post-test to one in which most students got 100%.

The results suggest that for difficult topics such as conditional probability, simply teaching procedural knowledge is not sufficient to help students achieve mastery of a topic. Rather, spending modest amounts of extra time also covering conceptual aspects of the topic such as how the topic differs from related topics or how to apply the topic to real-world problems can produce a very large increase in problem-solving performance. These results also suggest that instruction based on Anderson's original ACT-R framework, which suggests that expert knowledge consists of proceduralizing facts, is not sufficient to create subject matter expertise. Students who were taught based on this type of procedural framework, in general, performed very poorly. Rather, these results suggest that Leddo et al.'s extension of the ACT-R framework more closely explains the acquisition of true expertise, as students who were also taught the conceptual aspects of conditional probability achieved perfect or near perfect scores on the post-test.

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Conclusion and Recommendations

It can be concluded that for math students who are being introduced to conditional probability, there is a preference for procedural plus conceptual teaching formats, for students performed with overall higher scores when conceptual instruction was added to procedural instruction. One explanation for this difference may be that conditional probability is a subject for which more study time improves performance. However, more study time alone seems unlikely to account for the increased performance since the amount of extra time was relatively small compared to the huge increase in resulting performance.

The testing of procedural or conceptual lesson plans is not limited to math and can be assessed on virtually any subject that can be taught to students. Generally speaking, STEM topics follow a progression in which students must master the basics before they can move on and tackle more complicated subjects. This raises the interesting question of how well does conceptual vs. procedural instruction prepare students to learn even more advanced topics? A study done by Nadhi&Jatisunda begins to answer this question. The researchers investigated students' understanding of fractions in elementary school at a conceptual vs a procedural level [6]. Their research found that students with both procedural and conceptual knowledge will be able to develop useful knowledge to learn math effectively. They conclude that the deep understanding of the basics of fractions in math gives the student the ability to be successful in higher levels. Further research could build on this finding.

To conclude, it must be stated that many questions still remain unanswered that would greatly improve students' educational experience: if all courses at an institution included a conceptual teaching format, would the majority of students learn and perform noticeably better? Are certain subjects easier to grasp when taught procedurally instead of conceptually? As the skill level of a student increases, are they less likely to prefer being taught procedurally or conceptually? This remains a very fertile area for further investigation.

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