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SECURITIZATION NON-FUNGIBLE TOKENS ON THE BLOCKCHAIN

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ABSTRACT

Non-Fungible Token (NFT) is regarded as one of the important applications of blockchain technology. In this article, we propose an asset-backed securities (ABS) scheme that splits the complete NFT into a certain number of units, which are shared by multiple participants. On the one hand, ABS plans to promise high-value and long-term investment returns by enhancing the market liquidity of NFTs. On the other hand, securitized NFTs can participate in De-Fi as an automated market maker (AMM), just like AMM in alternative tokens. However, when a participant with a portion of the NFT tries to obtain full ownership of the NFT, the acquisition process may face some obstacles, including strategic bidding. Therefore, we proposed a game theory model and de- signed a novel NFT repurchase mechanism to overcome these obstacles. Our solution helps to successfully carry out the repurchase process at a reasonable price when issuing single-chip NFT asset-backed securities.

Keywords: Non-Fungible Token Game Theory Asset-Backed Securities Blockchain

Introduction

Since the birth of the first non-homogeneous token (NFT) [13], the world has witnessed an exponential increase in its popularity. Opensea [1] and other NFT markets are booming. The total number of NFTs on the platform exceeds 34 million, and the total transaction volume exceeds US4*billion.*

The technology of NFT is also developing rapidly. The first standard of NFT, ERC-721 [6] only supports a single type of non-homogeneous tokens. But now, ERC-1155 [5] can provide support for fungible and non-fungible tokens. Although the early NFT smart

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contracts were deployed on permission-free blockchains, there are now many NFT designs for permissioned blockchains [7].

However, the application of NFT still faces many obstacles. First of all, NFT pricing is a very immature function and lacks practical algorithms. Second, the value of some NFTs is extremely high, leading to their low market liquidity. Third, NFT is not fully compatible with the existing De-Fi [15] ecosystem, such as Oracles [9] and AMMs [2]. Fourth, NFT investment with a long payback period has a high risk. Finally, NFT assets like patents still need financial support to facilitate the development process, which requires a means of attracting funds, which is impossible because NFTs do not allow shared ownership.

There are many related studies trying to design a reasonable and complete repurchase agreement, regardless of whether the agreement is designed for the stocks of a specific company or other forms of securities. [8]. Most research has focused on repurchasing shares from shareholders. In the typical stock repurchase model [4], the company tries to repurchase a part of the stock from shareholders, the company announces a new investment, and sells the debt of the investment in the form of auction. And [11] is a blockchain solution based on repurchase.

The settings of these works cannot be directly applied to the out theme, because the financial ecology on the blockchain is very different from traditional finance.

Main Contributions

Our contribution is mainly reflected in two parts, the NFT securitization plan and the repurchase game.

NFT Securitization Scheme. We designed a smart contract that includes two types of NFTs, Complete NFT and Securitized NFT. Complete NFT is a general NFT like ERC-721. Securitized NFT is an asset-backed securities (ABS) issued by Complete NFT. We designed the process of securitizing a complete NFT into a securitized NFT and reconstructing the complete NFT from the corresponding securitized NFT.

The creation of securitized NFT managed to solve most of the problems faced by current NFT applications: Compared with the complete NFT counterpart, the value of securitized NFT is much lower, thereby increasing market liquidity; securitized NFT can be used as a Replaceable tokens to solve the problem of incompatibility with the De-Fi ecosystem; investment risk is greatly reduced; because multiple securitized NFTs will represent a complete NFT, and these securitized NFTs may belong to different owners, financing become possible.

As far as we know, ABSNFT is the first NFT solution to securitize NFT, and it has the ability to reconstruct it into a complete NFT after securitization.

Repurchase Scheme. There are still two problems with the NFT securitization program. First, it is difficult to collect all $SNFT\ (id)$ through pure market behavior. Second, there is still a lack of proper NFT pricing algorithms.

In order to solve these two problems, we designed a new NFT repurchase scheme based on Stackelberg Game [12]. The SNFT (id) repurchase game can be triggered by participants who hold more than half of SNFT (id). We analyzed the Stackelberg equilibrium (SE) in three different settings and obtained beautiful theoretical results. In the setting of a two-player single-round game, we prove that in SE, the buyback will give a price equal to its own value on SNFT (id). And all SNFT (id) will eventually go to the player with the higher value of SNFT (id) in the two-player repeated game. Finally, in the setting of a multiplayer single-round game, the cooperation of players does not bring higher utility.

We also discussed the setting of budget limits. We have proposed a solution that allows participants to conduct similar financing operations in transactions. Finally, we propose two solutions for players who may not bid in the game. These solutions can prevent the game process from being blocked and protect the effectiveness of lazy bidders

The rest of the paper is organized as follows. Section 2 introduces the NFT securitization plan. In Section 3 and Section 4, we studied single-round and repeated two-person buyback games. In Section 5, we analyzed the buyback game between multiple leaders and one follower. In the last section, we discussed solutions to address budget constraints and lazy bidders in a blockchain setting.

NFT Securitization Scheme

In this section, we would like to introduce the general framework of the smart contract for NFT, denoted by C_{NFT}.

As we know, fungible tokens are usually used as currency in blockchain sys- tem. Those tokens may be original tokens in blockchain system like ETH [14], or may be issued by smart contracts, such as stable coins [10]. For the sake of simplicity, we assume that all transactions in blockchain system are paid in one kind of unified fungible tokens. Such an assumption is reasonable because the exchange between fungible tokens are convenient, so that our setting for NFT can be easily extended to more general case. Moreover, we ignore the unit of fungible token, and thus directly use numbers to represent the quantity of them.

Basic Setting of NFT Smart Contract

There are two kinds of NFTs are discussed in this paper.

- **Complete NFT**. Complete NFTs are traditional non-fungible tokens, which appear in blockchain system as a whole. Each complete NFT has a unique token ID. We use $CNFT\,(id)$ to denote one complete NFT with token ID id.

- **Securitized NFT**. Securitized NFTs are the *A*sset Based Securities of complete NFTs. A complete NFT may be securitized into an amount of securitized units. A unit securitized NFT has an ID, denoted by $SNFT\ (id)$, which is associated to $CNFT\ (id)$. Unless the repurchase process is triggered, all securitized NFTs can be freely traded.

we assume that all complete NFTs and securitized NFTs belong to one same smart contract, denoted by C_{NFT}. Although the securitized NFTs are similar to the fungible tokens in ERC-1155 standard, our C_{NFT} is actually quite different from ERC-1155 standard [5]. That is because all securitized NFTs in C_{NFT}, associated to one complete NFT, have the same ID, while different NFTs or different fungible tokens generally have different token IDs in ERC-1155 standard. Therefore, we require that C_{NFT} is based on ERC-721 standard [6], and the complete NFTs are just the NFTs defined in ERC-721. Table 1 lists all functions in C_{NFT}.

The task of smart contract C_{NFT} includes securitizing complete NFTs, trading the securitized NFTs among participants, and restructing complete NFT after repurchasing all securitized NFTs with the same ID. Because the transactions of securitized NFTs are similar to those of fungible tokens, we omit the trading process here and introduce NFT securitization process, NFT repurchase process and NFT restruction process in subsequent three subsections respectively.

Table 1. The key functions of C*NFT*

Function Name	**Function Utility**
CNFT owner Of (id)	Return the address of the owner of CNFT (id).

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CNFT transfer From (addr1, addr2, id)	Transfer the ownership of CNFT (id) from address addr1 to address addr2. Only the owner of CNFT (id) has the right to trigger this function.
SNFT total Supply(id)	Return the total amount of SNFT (id) in contract C*NFT* .
SNFT balance Of (addr, id)	Return the amount of SNFT (id) owned by address addr.
SNFT transfer From (addr1,addr2, id, amount)	Transfer the ownership of amount unit of SNFT (id) from address addr1 to address addr2. Only the owners having some amount units of SNFT (id) can trigger this function.
CNFT securitization (addr, id, amount)	Freeze CNFT (id), and then transfer amount units of SNFT (id) to address addr. Only the owner of CNFT (id) can trigger this function.
CNFT restruction (addr, id)	Burn all SNFT (id), unfreeze CNFT (id), and then transfer the ownership of CNFT (id) to address addr. Only the one who owns all amounts of SNFT (id) can trigger this function.
Repurchase(id)	Start the repurchase process of SNFT (id). Only the one who owns more than half amounts of SNFT (id) can trigger this function.

NFT Securitization Process

In this subsection, we shall emphasize the issue of Asset Backed Securities for Complete NFTs.

Algorithm 1 detaily presents the NFT securitization process. To be specific, once the owner of *CNFT* (*id*) triggers *CNFT securitiation* (*addr, id, amount*), the *amount* units of securitized NFTs are generated and transferred to address *addr* in Line 2-4; and then the ownership of *CNFT* (*id*) would be transferred to a fixed address *FrozenAddr* in Line 5.

It is worth to note that if *Repurchase*(*id*) has not been triggered, securitized NFTs are freely traded in blockchain system.

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Algorithm 1 NFT Securitization

```
procedure CNFTsecuritization                                ▷ Triggered by sender
    require(sender == CNFTownerOf(id))  ▷ sender is the owner of CNFT(id)
    totalSupply[id] ← amount    ▷ Record the total amount of units of SNFT(id)
    tokenBalance[id][addr] ← amount          ▷ the amount units SNFT(id) are
  generated and transferred to address addr
    CNFTtransferFrom(sender, FrozenAddr, id)                ▷ Freeze CNFT(id)
```

NFT Repurchase Process

To realize the repurchase process efficiently, the repurchase mechanism is crucial. Before presenting the repurchase mechanism, we shall introduce some necessary notations.

After the securitization process, a complete NFT with ID $CNFT(id)$ is securitezed into M units of $SNFT(id)$. Suppose that there are $k+1$ participants, $N = \{N_0, \ldots\ldots N_k\}$, each owning m_i units of $SNFT(id)$. Thus $\Sigma^{k}_{i=0} m_i = M$.

If there is one participant, denoted by N_0, having more than half of $SNFT(id)$, then he can trigger the repurchase process and trade with each N_i, $i = 1, \cdots, k$. Let v_i be N_i's value estimate for one unit of $SNFT(id)$ and p_i be the price bidded by N_i, $i = 0, \cdots, n-1$, in a deal. Here our smart contract C_{NFT} requires each value $v_i \in \{1, \cdots\}$ and price $p_i \in \{0, 1, \ldots\}$ to discretize the analysis.

Mechanism 1 (Repurchase Mechanism) *For the repurchase between* N_0 and N_i, i = 1, · · · , k,

– if $p0 \geq pi$, then N_0 shall buy mi units of SNFT(id) from N_i at the price of $\frac{p0+p1}{2}$

– if $p0 \leq pi-1$, then N_i shall buy mi units of SNFT(id) from N_0 at the price of $\frac{p0+p1}{2}$

From Mechanism 1, we can see that the repurchase process only happens between N_0 and N_i, $i = 1, \cdots, n-1$. Particularly, once N_0 successfully repurchases m_i units of $SNFT(id)$, the utilities of N_0 and N_i are

$$U_0^i(p_0, p_i) = m_i(v_0 - \frac{p_0 + p_i}{2}),\ U_i(p_0, p_i) = m_i(\frac{p_0 + p_i}{2} - v_i),\ if\ p_0 \geq p_i. \quad (1)$$

However, if N_0 fails to repurchase from N_i, then N_i shall buy m_i units of $SNFT(id)$

from N_0 at the cost of $m_i\frac{p_0+p_i}{2}$, while N_0 only obtains a discounted revenue $m_i\frac{p_0+p_i-1}{2}$ to punish its failure. So the utilities of N_0 and N_i are

$$U_0^i(p_0,p_i) = m_i\left(\frac{p_0+p_i-1}{2} - v_0\right),\ U_i(p_0,p_i) = m_i\left(v_i - \frac{p_0+p_i}{2}\right),\ if\ p_0 \le p_i - 1 \quad (2)$$

During the repurchase process, the key issue for each participant is how to bid the price p_i, $i = 0, , k$, based on its own value estimate. To solve this issue, we would model the repurchase process as a stackelberg game to explore the equilibrium pricing solution in the following Section 3 to 5.

NFT Restruction Process

Once one participant successfully repurchases all securitized NFTs, he has the right to trigger *CNFTrestruction*(*addr,id*), shown in Algorithm 2, to burn these securitized NFTs in Line 3 to 4 and unfreeze *CNFT* (*id*), such that the ownership of *CNFT* (*id*) would be transferred from address *FrozenAddr* to this participant's address *addr* in Line 5.

After NFT restruction, all *SNFT* (*id*) are burnt, and *CNFT* (*id*) is unfrozen. Hence, the owner of *CNFT* (*id*) has the right to securitize it or trade it as a whole.

Algorithm 2 NFT Restruction

1: **procedure** CNFTRESTRUCTION ▷ Triggered by *sender*
2: $require(tokenBalance[id][sender] == totalSupply[id])$ ▷ *sender* should be the owner of all $SNFT(id)$
3: $totalSupply[id] \leftarrow 0$ ▷ Burn all $SNFT(id)$
4: $tokenBalance[id][sender] \leftarrow 0$ ▷ Burn all $SNFT(id)$
5: $CNFTtransferFrom(FrozenAddr, addr, id)$ ▷ Unfreeze $CNFT(id)$

3. Two-Player Repurchase Stackelberg Game

In this section, we discuss the repurchase process for a two-player scenario. To be specific, in the two-player scenario, when a player owns more than half of SNFT (id), denoted by N_0, he will trigger the repurchase process with another player N_1. To explore the optimal pricing strategy for both of players, we model the repurchase process as a two-stage Stackelberg game, in which N_1 acts as the leader to set its price p_1 in Stage I, and N_0, as the follower, decides its price p_0 in Stage II. Recall all prices and all values are in $\{0, 1, \cdots\}$.

(1) *N_0's pricing strategy in Stage II:* Given the price of p_1, set by N_1 in Stage I,

participant N_0 decides its price to maximize its utility, which is given as:

$$U_0(p_0,p_1)=\begin{cases} m_1(v_0-\frac{p_0+p_1}{2}) & if\ p_0\geq p_1; \\ m_1(\frac{p_0+p_1-1}{2}-v_0) & if\ p_0\leq p_1-1. \end{cases} \quad (3)$$

(2) N_1's pricing strategy in Stage I: N_1 determines the optimal price for maxi-mizing its utility as:

$$U_1(p_0,p_1)=\begin{cases} m_1(\frac{p_0+p_1}{2}-v_1) & if\ p_0\geq p_1; \\ m_1(v_1-\frac{p_0+p_1}{2}) & if\ p_0\leq p_1-1. \end{cases} \quad (4)$$

Analysis of Stackelberg Equilibrium

(1) Best response of N_0 in Stage II. Given the price p_1 provided by N_1, in Stage II, N_0 shall determines its best response $BR_2(p_1)$ to maximize its utility.

Lemma 1. In the two-stage Stackelberg game for repurchase process, if the price p_1 is given in Stage I, the best response of N_0 in Stage II is

$$BR_2(p_1)=\begin{cases} p_1-1 & if\ p_1\geq v_0+1 \\ p_1 & if\ p_1\leq v_0 \end{cases} \quad (5)$$

Proof. According to (3), U_0 is monotonically increasing when $p_0 \leq p_1 - 1$ and monotonically decreasing when $p_0 \geq p_1$. So $BR_2(p_1) \in \{p_1 - 1, p_1\}$. In addition, when $p_1 \geq v_0 + 1$, we have

$$U_0(p_0 = p_1, p_1) = m_1(v_0 - p_1) < 0 \leq m_1(p_1 - 1 - v_0) = U_0(p_0 = p_1 - 1, p_1).$$

It implies that the best response of N_0 is $BR_2(p_1) = p_1 - 1$ if $p_1 \geq v_0 + 1$. When

$p_1 \leq v_0$, we have

$$U_0(p_0 = p_1, p_1) = m_1(v_0 - p_1) \geq 0 > m_1(p_1 - 1 - v_0) = U_0(p_0 = p_1 - 1, p_1).$$

So under the situation of $p_{0\leq}v_0$, the best response of N_0 is $BR_2(p_1) = p_1$. This lemma holds.

(2) The optimal strategy of N_1 in Stage I. The leader N_1 would like to optimize its pricing strategy to maximize its utility shown in (4).

Lemma 2. In the two-stage Stackelberg game for repurchase process, the opti- mal pricing strategy for the leader N_1 is

$$p_1^* = \begin{cases} v_0 & if\ v_0 \geq v_1 \\ v_0 + 1 & if\ v_0 \leq v_1 - 1. \end{cases} \qquad (6)$$

Proof. Based on Lemma 1, we have

$$U_1(BR_2(p_1), p_1) = \begin{cases} m_1(p_1 - v_1) & if\ p_1 \leq v_0; \\ m_1(v_1 - p_1 + \frac{1}{2}) & if\ p_1 \geq v_0 + 1. \end{cases}$$

when $p_1 \geq v_0 + 1$, indicating the optimal pricing strategy $p_1^* \in \{v_0, v_0 + 1\}$. In addition, for the case of $v_0 \geq v_1$, if $p_1 = v_0$, then $p_0^*(p_1) = p_1 = v_0$ by Lemma 1 and $U_1(v_0, v_0) = m_1(v_0 - v_1) \geq 0$. On the other hand, if $p_1 = v_0 + 1$, then $p_0^*(p_1) = p_1 - 1 = v_0$ by Lemma 1 and $U_1(v_0, v_0 + 1) = m_1(v_1 - v_0 - \frac{1}{2}) < 0$. Therefore, $U_1(v_0, v_0) > U_1(v_0, v_0 + 1)$, showing the optimal pricing strategy of N_1 is $p_1^* = v_0$ when $v_0 \geq v_1$. Similarly, for the case of $v_0 \leq v_1 - 1$, we can conclude that $p_1^* = v_0 + 1$. This lemma holds.

Combining Lemma 1 and 2, the following theorem can be derived directly.

Theorem 1. When $v_0 \geq v_1$, there is exactly one Stackelberg equilibrium where $p_1 = p_0 = v_0$. And when $v_0 \leq v_1 - 1$, there is exactly one Stackelberg equilibrium where $p_0 = v_0$, $p_1 = v_0 + 1$.

Furthermore, the following theorem demonstrates the relation between Stackelberg equilibrium and Nash equilibrium.

Theorem 2. Each Stackelberg equilibrium in Theorem 1 is also a Nash equilibrium.

Proof. From Theorem 1 we know that the best response of N_0 is always $BP_0 = v_0$. Next, we shall discuss the best response of N_1 under the condition that N_0's pricing strategy is $p_0 = v_0$. By (4), we have

$$U_1(v_0, p_1) = \begin{cases} m_1(\frac{v_0+p_1}{2} - v_1) & if\ p_1 \leq v_0; \\ m_1(v_1 - \frac{v_0+p_1}{2}) & if\ p_1 \geq v_0 + 1. \end{cases}$$

So U_1 monotonically increases when $p_1 \leq v_0$ and monotonically decreases when $p_1 \geq v_0 + 1$, implying $p_1^* \in \{v_0, v_0 + 1\}$. Particularly, when $v_0 \geq v_1$, we have

$U_1(v_0, v_0) = m_1(v_0 - v_1) \geq 0 > m_1(v_1 - v_0 - \frac{1}{2}) = U_1(v_0, v_0 + 1)$, showing the best response of N_1 is $p_1^* = v_0$. On the other hand, when v_0 v_1 1, we have

$U_1(v_0, v_0) = m_1(v_0 - v_1) < 0 < m_1(v_1 - v_0 - \frac{1}{2}) = U_1(v_0, v_0 + 1)$, showing the best response of N_1 is $p_1^* = v_0 + 1$. This result holds.

3.2 Analysis of Bayesian Stackelberg Equilibrium

In previous subsection, the Stackeberg equilibrium Is deduced based on the com- plete information about the value estimate v_i, $i = 0, 1$. However, the value estimates may be private in practice, which motivates us to study the Bayesian Stackelberg game with incomplete information. In this proposed game, although the value estimate v_i is not known to others, except for itself N_i, $i = 0, 1$, the probability distribution of each V_i is public to all. Here we use V_i to denote the random variable of value estimate. Based on the assumption that all V_i are integers in our smart contract, we continue to assume that each N_i's value estimate V_i has finite integer states, denoted by $v^1, v^2, \cdots, v^{k_i}$, and its discrete probability distribution is

$Pro(V_i = v_i^l) = P_i^l$, $l = 1, \cdots, k_i$, and $\sum_{l=1}^{k_i} P_i^l = 1$.

(1) Best response of N0 in Stage II. Because v0 is deterministic to N0, and p1 is given by N_1 in Stage I, Lemma 1 still holds, so

$$BR_2(p_1) = \begin{cases} p_1 - 1 & if\ p_1 \geq v_0 + 1; \\ p_1 & if\ p_1 \leq v_0. \end{cases}$$

(2) Optimal pricing strategy of N_1 in Stage I. According to Lemma 1, we have

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$$U_1(BR_2(p_1), p_1) = \begin{cases} m_1(p_1 - v_1) & if\ p_1 \leq v_0; \\ m_1(v_1 - p_1 + \frac{1}{2}) & if\ p_1 \geq v_0 + 1. \end{cases}$$

Based on the probability distribution of V0, the expected utility of U1 is:

$$E_1(p_1) = \sum_{v_0^l \geq p_1} m_1(p_1 - v_1)P_0^l + \sum_{v_0^l \leq p_1 - 1} m_1(v_1 - p_1 + \frac{1}{2})P_0^l. \qquad (7)$$

Let us compute the first derivative of (7), and obtain

$$\frac{dE_1(p_1)}{dp_1} = m_1(\sum_{v_0^l \geq p_1} P_0^l - \sum_{v_0^l \leq p_1 - 1} P_0^l).$$

Since $\sum_{v_0^t \geq p1} P_0^1$ decreases with p1 and $\sum_{v_0^t \geq pI-1} P_0^1$ increases with p1,$\frac{dE_1\,(p_1)}{dp_1}$ monotonically decreases with p1, showing E1(p1) is concave and has an optimal price P_1^*, such that $P_1^* = argmax_{p1}\, E_1(p_1).\sum_{v_0^t \geq p1} P_0^1$

Theorem 3. There is a Stackelberg equilibrium in Bayesian Stackelberg game.

(1) If $P_1^* \leq v_0$, then $p_0 = P_1^*$ and $p_1 = P_1^*$ is a Stackelberg equilibrium.
$*$ $*$

(2) If $p_1 \geq v_0 + 1$, then $p_0 = p_1 - 1$ and $p_1 = p_1$ is a Stackelberg equilibrium.

4 Repeated Two-Player Stackelberg Game

In this section, we would extend the study of one-round Stackelberg game in previous section to the repeated Stackelberg game. Before our discussion, we shall construct the basic model of repeated two-player Stackelberg game by introduce some necessary notations.

Definition 1. Repeated two-player Stackelberg repurchase game is given by a tuple G_r = (M, N, V, S, L, P, U), where:

- N = { N_0, N_1} is the set of players. The role of being a leader or a follower may change I the whole repeated process.
- M is the total amount of SNFT(id) . W.l.o.g , We assume that M is odd, such that one of {N_0 , N_1} must have more than half of SNFT(id).

- V= {v_0 , v_1} is the set of value estimate by players . Let $v_i \in \{1,2,3,\dots.\}$.
- S= {s^1 , s^2 , …….s^t , z} is the set of sequential states . $s^j = (m_0^j, m_1^j)$, in which $m_0^j, m_1^j > 0, m_0^j + m_1^j = M$, and $m_0^j \neq m_1^j$ because M is odd. Z_1 = (0,M). zεZ = {Z_0,, Z_1} represents the terminal state , where Z_0 = (M_0 , 0), z_1 = (0, M). If the sequential states are infinity , then t= +∞. Let us denote (m_0^{t+1}, m_1^{t+1}) =z.
- L= {l^1 ,l^2 , ….., l^t} is the set of sequential leaders. To be specific , $l^i = N_0$, if $m_0^j > m_1^j$; otherwise , $l^i = N_1$.
- $P_i = \{p_i^1, p_i^2, \dots.., p_i^t\}$ is the set of sequential prices given by N_i , $p_i^j \in \{0,1,2,..\}$.
- $U_i : S \times P_0 \times P_1 \rightarrow R$ is the utility function for player N_i in a single round.

The concrete expressions of Ui will be proposed latter.

Repeated Stackelberg Game Procedure Repeated game Gr is consist of several rounds, and each round contains two stages. In the j-th round,

- In **Stage I**, the leader provides a price $p_l^j \in \{0, 1, \cdots\}$.
- In **Stage II**, the follower provides a price $p_{1-l}^j \in \{0, 1, \cdots\}$.
- If $p_l \leq p_{1-l}$, N_{1-l} successfully purchased m_l^j units of $SNFT(id)$ from N_l at the total cost of $m_l^j \frac{p_l^j + p_{1-l}^j}{2}$. And the price of each unit $SNFT(id)$ is $\frac{p_l^j + p_{1-l}^j}{2}$.
- If $p_l \geq p_{1-l} + 1$, N_l purchases m_l units of $SNFT(id)$ from N_{1-l} at the total cost of $m_l^j \frac{p_l^j + p_{1-l}^j}{2}$. And the price of each unit $SNFT(id)$ is $\frac{p_l^j + p_{1-l}^j}{2}$.

The whole game process is shown in Figure ??. Based on the description for the j-th round of repeated game, the utilities of N0 and N1 are

$$U_0(m_0^j, m_1^j, p_0^j, p_1^j) = \begin{cases} (v_0 - (p_0^j + p_1^j)/2)m_1^j & if\ p_0^j \geq p_1^j, m_0^j > m_1^j; \\ ((p_0^j + p_1^j - 1)/2 - v_0)m_1^j & if\ p_0^j < p_1^j, m_0^j > m_1^j; \\ ((p_0^j + p_1^j)/2 - v_0)m_0^j & if\ p_1^j \geq p_0^j, m_0^j < m_1^j; \\ (v_0 - (p_1^j + p_0^j)/2)m_0^j & if\ p_1^j < p_0^j, m_0^j < m_1^j; \end{cases} \quad (8)$$

$$U_1(m_0^j, m_1^j, p_0^j, p_1^j) = \begin{cases} ((p_0^j + p_1^j)/2 - v_1)m_1 & if\ p_0^j \geq p_1^j, m_0^j > m_1^j; \\ (v_1 - (p_0^j + p_1^j)/2)m_1^j & if\ p_0^j < p_1^j, m_0^j > m_1^j; \\ (v_1 - (p_0^j + p_1^j)/2)m_0 & if\ p_1^j \geq p_0^j, m_0^j < m_1^j; \\ ((p_0^j + p_1^j - 1)/2 - v_1)m_0 & if\ p_1^j < p_0^j, m_0^j < m_1^j. \end{cases} \quad (9)$$

Each player is interested in its total utility in the whole process

$$U_i = \sum_{j \in \{1,2,\cdots,t\}} U_i(m_0^j, m_1^j, p_0^j, p_1^j).$$

Lemma 3. For each player N_i, i ∈ {0, 1}, if its price is set as $p_i^j = V_i$ in the j-th round, j∈ {1,2,…t}, then $U_i\,(m_0^j, m_1^j, p_0^j, p_1^j) \geq 0$

This result can be directly deduced from (8) and (9).

Lemma 4. If the repeated game goes through indefinitely, that is t = +∞, then U0 + U1 = −∞.

Proof. For the j-th round, let $N_1 = l^j$ be the leader and thus N_{1-l} is the follower. Since there are only two players, all SNFT(id) will belong to one player, if the follower can successfully repurchase SNFT(id) from the leader, and then the repeated game stops. It means that in the j-th round, m_l^j units of SNFT(id) is bought by N_{1-c} from N_l and the game stops at the terminal state Z_{1-l}. So if the repeated game goes through indefinitely, it must be that $P_l^j > P_{1-l}^j$, for each j ∈ {1, 2, ・・・}. Then we have

$$U_0(m_0^j, m_1^j, p_0^j, p_1^j) + U_1(m_0^j, m_1^j, p_0^j, p_1^j) = (m_0^{j+1} - m_0^j)(v_0 - v_1) - \frac{1}{2}m_c^j;$$
$$U_0(m_0^j, m_1^j, p_0^j, p_1^j) + U_1(m_0^j, m_1^j, p_0^j, p_1^j) \leq (m_0^{j+1} - m_0^j)(v_0 - v_1) - \frac{1}{2};$$

And

$$\begin{aligned} U_0 + U_1 &= \sum_{j=\{1,2,\cdots,t\}} U_0(m_0^j, m_1^j, p_0^j, p_1^j) + U_1(m_0^j, m_1^j, p_0^j, p_1^j) \\ &\leq \sum_{j=\{1,2,\cdots,t\}} (m_0^{j+1} - m_0^j)(v_0 - v_1) - \frac{1}{2} \\ &= (m_0^{t+1} - m_0^1)(v_0 - v_1) - \frac{1}{2}t \leq M|v_0 - v_1| - \frac{1}{2}t = -\infty. \end{aligned}$$

This result holds.

Combining Lemma 3 and Lemma 4, we have the following conclusion.

Lemma 5. If there is a Stackelberg equilibrium in the two-player repeated Stackelberg game, then $U_1 + U_2 \geq 0$ in this Stackelberg equilibrium.

Proof. Suppose to the contrary that U1+U2 < 0 in this Stackelberg equilibrium, then there must exist i ∈ {0, 1}, such that Ui < 0. However, by Lemma 3, we know that if each player sets its price as $p_i^j = v_i$, then its utility $u_i^j \geq 0$ Hence N_i can obtain more utility by setting $p_i^j = v_i$ which is a contradiction that N_i doesn't give a best response in this Stackelberg equilibrium.

Combining Lemma 4 and Lemma 5, we have

Lemma 6. If there is a Stackelberg equilibrium in the two-player repeated Stackelberg game, then the repeated game stops in a finite number of steps, meaning $t < +\infty$, in this Stackelberg equilibrium.

The following theorem states that once a Stackelberg equilibrium exists and $v_i > v_{1-i}$, then this player Ni must buy all SNFT(id) at last.

Theorem 4. If vi > v_{1-i} and a Stackelberg equilibrium exits, then $z = z_i$, in all Stackelberg equilibria.

Proof. By (8) and (9) we have

$$U_0(m_0^j, m_1^j, p_0^j, p_1^j) + U_1(m_0^j, m_1^j, p_0^j, p_1^j) \leq (m_0^{j+1} - m_0^j)(v_0 - v_1).$$

$$\begin{aligned} U_0 + U_1 &\leq \sum_{j \in \{1,2,\cdots,t\}} u_0(m_0^j, m_1^j, p_0^j, p_1^j) + u_1(m_0^j, m_1^j, p_0^j, p_1^j) \\ &\leq \sum_{j \in \{1,2,\cdots,t\}} (m_0^{j+1} - m_0^j)(v_0 - v_1) = (m_0^{t+1} - m_0^1)(v_0 - v_1). \end{aligned}$$

If $v_0 > v_1$, then it must be $m_0^{t+1} > m_0^1$. Otherwise, $U_0 + U_1 < 0$. It's a contradiction. In addition, because $m_0^{t+1} \in \{0, m\}$ and $m_0^1 > 0$, we have m_0^{t+1} = M. Therefore, at last $z = z_0$. Similarly, it is easy to deduce $z = z_1$ if $v_1 > v_0$.

Based on Theorem 4, we can explore the Stackelberg equilibrium of the twoplayer repeated

Stackelberg game in the following theorem, whose proof is provided in Appendix A.

Theorem 5. If $v_i > v_{1-i}$, the following strategy is a Stackelberg equilibrium:

$$p_{1-i}^j = v_{1-i}; \quad p_i^j = \begin{cases} v_{1-i}+1 & if\ l^j = i; \\ p_{1-i} & if\ l^j = 1-i, p_{1-i} \le v_{1-i}; \\ p_{1-i}-1 & if\ l^j = 1-i, p_{1-i} > v_{1-i}. \end{cases} \qquad (10)$$

5 Multi-Player Repurchase Stackelberg Game

In Section 4, we model a two-stage Stackelberg game to study the repurchase process for the two-player scenario. In this section, we would extend the discussion for multi-player scenario, in which N_0 has more than half of S_{NFT} (id), and $\{N_1,\ldots,N_k\}$ are repurchased players. To be specific, N_0 triggers the repurchase process, and asks all other repurchased players $\{N_1,\ldots,N_k\}$ to bid their prices pi at first, and N_0 decides its price p_0 later. So, we also model the repurchase process in multi-player scenario as a two-stage Stackelberg game, where $\{N_{1,\ldots\ldots},N_k\}$ are the leaders to determine their prices in Stage I, and N_0 acts as the followers to decide its price p_0 in Stage II. Different with the two-player scenario, N_0 shall trade with each N_i, i = 1,…, k, in the multi-player scenario.

Then each Ni, i = 1,…., k, has its utility $U_i(p_0, p_i)$ as (1) and (2). But the utility of N_0 is the total utility of N_0 from the trading with each N_i. That is

$$U_0(p_0, p_1, \cdots, p_k) = \sum_{i=1}^{k} U_0^i(p_0, p_i),$$

Where u_0^i (p_0 , p_1) is defined as (1) and (2). The multi-player Stackelberg repurchase game is illustrated in Figure 1.

5.1 Analysis of Stackelberg Equilibrium

In the Stackelberg repurchase game for multi-player scenario, N_0 shall trade with each N_i, i = 1,… k. Inspired by the Stackelberg equilibrium in two player Stackelberg game, we first discuss the best response of N_0, if each Ni bids its price as

$$p_i^* = \begin{cases} v_0 & if\ v_i \le v_0; \\ v_0+1 & if\ v_i \ge v_0+1. \end{cases} \qquad (11)$$

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Then we study the collusion from a group of repurchased players. Our task is to prove that once a group of repurchased players deviate the pricing strategy (11), then their total utility must be decreased. This guarantees that each repurchased player would like to follow the pricing strategy (11).

Lemma 7. In the Stackelberg repurchase game for multi-player scenario, if all leaders set their prices { p_i^*} as (11) in Stage I, then the best response of the follower N_0 in Stage II is BR ($p_1^* \ldots p_n^*$) = v_0.

Proof. For each trading between N_0 and N_i, i = 1,….,k, Lemma 1 ensures that v_0 = argmax$_{p0}$ u_0^i (p_0 , p_i^*). Since each u_0^i (p_0 , p_i^*) ≥ 0, we have

$$BR_2(p_1^*, \cdots, p_k^*) = argmax_{p_0} U_0(p_0, p_1^*, \cdots, p_k^*) = argmax_{p_0} \sum_{i=1}^{k} U_0^i(p_0, p_i^*) = v_0.$$

This lemma holds.

To study the collusion of repurchased players, we partition the set of {N_1,….,N_k}into two disjoint subsets A and B, such that each $N_i \in$ A follows the pricing strategy (11) while each $N_i \in$ B does not. Thus given all prices provided by players, the price profile p = (p_0 {p_i^*}$N_{i\in}$A, {p_i}N_i $_\in$ B) can be equivalently expressed as p = (p_0 , p_A^* ,PB). Here we are interested in the total utility of all players in B, and thus define

$$U_B(p_0, \mathbf{p}_A^*, \mathbf{p}_B) = \sum_{N_i \in B} U_i(p_0, p_i).$$

Then we have the following Lemma, which shows that once a group of players deviate the pricing strategy (11), then their total utility will decrease. We move the proof to Appendix B.

Lemma 8. Let A = {N_i |P_i = p_i^* } and B= { N_i | $P_i \neq p_i^*$ }. Then U_B (BR_2 (p_A^* , PB) <U_B (v_0 ,$p_1^*, p_2^* \ldots, p_k^*$).

Theorem 6. In the multi-player Stackelberg repurchase game, the price profile ($P_0, P_1^*, \ldots \ldots, P_k^*$) is a Stackelberg equilibrium, where P_i^* is set as (11).

Proof. To simplify our discussion, we define the price profile $P^* = (P_1^*, \ldots \ldots, P_k^*)$, and P_{-1} denotes the profile without the price of N_i . So, $P^* = (P_{-i}^*, P_i^*)$. From Lemma 7, we have the best response of N_0 in Stage II is BR_2 (P^*) = v_0 . On the other hand, Lemma 8 indicates that no one would like to deviate the pricing strategy (11), since

$$U_i(BR_2(\mathbf{p}^*_{-i}, p_i), \mathbf{p}^*_{-i}, p_i) < U_i(v_0, \mathbf{p}^*_i).$$

Thus given the price profile P^* nobody would like to change its strategy P_i^* unilaterally. Therefore,$(P_0 , P_1^*, \dots \dots , P_k^*)$ is a Stackelberg equilibrium.

From the perspective of cooperation, we can observe that no group of repurchased players would like to collude to deviate the pricing strategy (11) by Lemma 8. Thus we have the following corollary.

Corollary 1. Given the Stackelberg equilibrium of $(P_0 , P_1^*, \dots \dots , P_k^*)$ no group of repurchased players would like to deviate this equilibrium.

6 Discussion

6.1 A Blockchain Solution to Budget Constraints

In previous settings, we don't consider the budget constraints. It's a common problem for many newly proposed mechanisms, but we still have a blockchain solution for budget constraints.

Our mechanism consists of two stages, N_0 gives price p_0 in the second stage, and other participants give prices in the first stage. Because N_0 wants to repurchase all SNFT(id), we think N_0's budget is no less than $(M – m_0)p_0$. So we ignore the budget constraint for N_0. For N_i that $i \neq 0$, if $P_i > P_0$, N_i Should Pay $\frac{P_0+P_i}{2}M_i$. We allow N_i to sell the chance of buying m_i pieces of SNFT(id) to anyone in the blockchain system. Specifically, after the second stage, we have another four stages to finish the payment procedure.

– Payment Stage One. N_0 pays $\sum_{i\in\{1,2,\cdots,k\},p_i>p_0} \frac{p_0+p_i}{2}m_i$. After the payment, N_0 gets $\sum_{i\in\{1,2,\cdots,k\},p_i>p_0} m_i$ pieces of $SNFT(id)$, $\forall i \in \{1, 2, \cdots , k\}$ and $p_i > p_0$, N_i gets $\frac{p_0+p_i}{2}m_i$ and loses m_i pieces of $SNFT(id)$.
– Payment Stage Two. For all $i \in \{1, 2, \cdots , k\}$ that $p_i > p_0$, N_i pays $\frac{p_0+p_i}{2}$ or gives a price $p^c_i \in \mathbb{Z}$. After the payment of $\frac{p_0+p_i}{2}$, N_0 gets $\frac{p_0+p_i-1}{2}$ and loses m_i pieces of $SNFT(id)$.
  p^c_i denotes the price of the chance of buying m_i pieces of $SNFT(id)$. If $p^c_i < 0$, p^c_i should pay $p^c_i < 0$ in this stage additionally. If N_i do nothing, we regard that N_i gives $p^c_i = 0$.

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– Payment Stage Three. For all $i \in \{1, 2, \cdots, k\}$ that gives a price p_i^c in Payment Stage Two, any participant N_i^c in the Blockchain system can propose a transation to pay $p_i^c + \frac{p_0+p_i}{2}m_i$. After the payment, N_0 gets $\frac{p_0+p_i-1}{2}m_i$ and loses m_i pieces of $SNFT(id)$, N_i^c gets m_i pieces of $SNFT(id)$. If $p_i^c > 0$, N_i gets p_i^c.
  Denote C as the set of $i \in \{1, 2, \cdots, k\}$ that gives a price p_i^c in Payment Stage Two, but there is no participants that pays $p_i^c + \frac{p_0+p_i}{2}m_i$ in this stage.
– Payment Stage Four. $\forall i \in C$, N_0 can choose to pay $2p_0 - p_i$. After the payment, N_0 gets m_i pieces of $SNFT(id)$, and N_i gets $2p_0 - p_i$ and loses m_i pieces of $SNFT(id)$

Our mechanism is a bit different if we add these four payment stages. And it's a conceptually novel solution towards the budget constraint problem. It's our future work to construct a model to analyse the repurchase scheme with the new payment procedure.

6.2 A Blockchain Solution to Lazy Bidders

Under extreme circumstances, some holders of the SNFT(id)s might not bid at the game. We name these participants as lazy bidders. To prevent the game process from being blocked and protect the utility of lazy bidders, we have the following two solutions.

– Custody Bidding. NFT's smart contract supports the feature for the NFT's owner to assign administrators who would have the authority over a series of NFT actions. The administrators could have the right to bid when the owner is idle and fails to make a bid. Players can also choose decentralized custody schemes [3] to host their Securitized NFT.

– Value Predetermination. Whenever a player obtains any pieces of SNFT(id), the player is required to predetermine the value at which he is willing to bid at and this information is stored in the smart contract. By the time the repurchase game initiates, if a player fails to make a bid within a certain amount of time, the smart contract automatically bids for the player with the predetermined price. This does not mean, however, that the player has to bid at the predetermined price if the player decides to make an active bid.

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