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IMPROVING SAT MATH SCORES BY USING METACOGNITIVE PROBLEM-SOLVING STRATEGIES

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ABSTRACT

Standardized testing is widely used in countries around the world and is often associated with high stakes as schools use test results in their admissions and graduation/certification decisions. Because of the importance of these tests, a plethora of test preparation services has arisen, many of which teach generalized test taking strategies. Given that research suggests that these programs produce modest improvement in SAT scores, the present study investigated whether teaching metacognitive math strategies would lead to improved scores on the math portion of the SAT test, which is widely used in college admissions decisions. An experiment was conducted in which students were either given concept/formula-based strategies that were published by the College Board, makers of the SAT test, for solving circle problems or metacognitive strategies that involved categorizing circle problem types by problem solving methods and then showing the methods for solving those types of problems. Students were then given a post-test that contained 20 circle problems and 10 non-circle geometry problems. The latter were included to test whether the problem-solving strategies students learned transferred to other types of problems. Results showed that students taught metacognitive problem-solving strategies performed significantly higher on the post-test than those who were taught using the College Board concept/formula-based strategies. This result held for both circle and non-circle problems suggesting that the metacognitive strategies were more effective than the concept/formula-based strategies and transferred to non-circle problems as well.

Introduction

Standardized testing is pervasive in education. Such testing is used to determine whether students have successfully mastered course material, graduate, or gain admission into schools. In

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the United States, for example, the SAT and ACT tests are weighed heavily as admission criteria for most colleges. As a result, test preparation has become a substantial industry, one that offers to help students increase their test scores.

Given the expense associated with test preparation classes, a major question arises as to how effective such test preparation services are. Indeed, there is some empirical evidence that most test preparation services produce very little effect. For example, Becker (1990) reports that test preparation typically raises SAT scores by only nine to 20 points, a negligible amount. Such nominal improvement may be attributable to the fact that many test preparation services focus largely on test taking skills such as time management and evaluating answer choices and less on core skills such as reading comprehension and mathematical concepts. This low increase in test scores is plausible since most test preparation courses are of limited duration, while reading, writing and mathematical skills are typically developed over a period of years.

The present research continues our previous research examining the question of whether SAT (and by extension, other standardized) test scores can be increased by teaching metacognitive strategies. Our previous research (Leddo et al., 2019) showed that using metacognitive reading strategies led to higher SAT reading scores than using standard reading and test-taking strategies advocated by the College Board and test preparation services. Similarly, we found that metacognitive writing strategies boosted SAT writing scores (Leddo et al., 2020).

In the present study, we explore whether using metacognitive math strategies can likewise boost SAT math scores. There already exists a substantial body of research that explores use of metacognitive problem-solving skills in mathematics (cf., Schneider and Artelt, 2010; Carr et al., 1994; Deoete and Veenman, 2006; Kramarski and Mevarech, 2003). Most of this research explores the relationship of metacognitive problem-solving skills in standard academic mathematics as opposed to standardized testing. Paek (2002) did examine the use of metacognitive problem solving in SAT mathematics problem solving and found that students who used metacognitive problem-solving skills scored higher than those who did not. However, Paek's research is correlational in nature, examining students' spontaneous use of metacognitive math strategies as opposed to whether teaching such skills could boost SAT math scores.

The present study explores the question of whether teaching students metacognitive problem-solving skills can boost SAT math scores when compared to teaching students a more traditional way. Since the term "metacognitive problem solving" encompasses a variety of strategies, we begin by operationalizing how we use the term in the present paper. Our previous research, referred to above, in metacognitive strategies applied to SAT reading and writing as well as research comparing how math practitioners solve practical problems drive the current use of the term "metacognitive problem solving."

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In our research comparing students to math practitioners (Leddo et al., 1992), we had both students and people who used math as part of their jobs (e.g., teacher, survey statistician, scientist) solve algebra word problems. We asked them to think aloud as they solved problems and recorded their problem-solving processes. The key difference between the two groups was that students' first step was invariably to list an algebraic formula and then look for numbers to plug into the formula while math practitioners would create a mental model of the problem (often referred to as "structuring the problem") and determine what information they needed to solve the problem. Only then did the practitioners look for a formula and numbers to enter into the formula. Often the formulas used by the practitioners were simpler than those used by students.

In many cases, both students and practitioners were able to solve the problems. While the practitioners never ran into trouble, students often did when they picked the wrong formulas or the wrong values from the problems to enter into the problems. When this happened, they often resorted to random operations on numbers from the problems, hoping that something would make sense.In order to induce students to spend time creating mental models of the problem, we created a technique we called "math without numbers."In this technique, students were given math problems without numbers, e.g., "You rent boats at a harbor. A couple wants to rent a boat and wants to know how long it will take them to reach the island. What information do you need to give them an answer?" When tested in a Colorado school, students using a curriculum based on this technique showed a 50% improvement in math problem solving performance (Leddo et al., 1992).

The idea that students' biggest need in mathematics problem solving is setting up the problem is reinforced by our previous work in applying metacognitive strategies to SAT reading and writing. In both of those studies, metacognitive strategies instruction focused on teaching students the types of material they might encounter on an SAT test. For reading, students were told the types of reading passages that appear on the SAT and how the writers organize the content. This enabled students to understand the information contained in each section of the passage as they were taught the writer's intent and structure for writing such a passage. For writing, students were taught the different types of editing questions they may encounter and strategies for addressing them. Key to these strategies was maintaining the flow of the SAT writing passages to highlight the points the writing passage writers were trying to make.

In both reading and writing studies, the metacognitive strategies ultimately centered on showing students different structures that writers use to convey concepts they are trying to communicate. By extension, the implication for SAT mathematics is to have metacognitive strategies that show the types of problems that can be used to convey underlying mathematical principles and formats. In this type of instruction, a student is taught the different problem structures and

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associated questions the problems ask and how the formulas they need to solve them operate in service of answering those questions. This may include supplementing formulas with general problem-solving strategies, as is discussed in the Method section. In many ways, this mirrors real-life applications of mathematics. For example, companies may want to find a price point that maximizes profits. An investor may want to have a certain monthly income at retirement and needs to decide how many to invest in different investment options. In both cases, math is used, but, in each case, math is the means for arriving at the solution not the end in itself.

This approach is the opposite of how the College Board itself prepares students for the SAT math section. In the College Board's own SAT preparation book, for each topic covered, a list of formulas or concepts is shown. These are followed by sample problems and the student is shown how to solve them using the formulas just covered. In other words, instruction is formula/concept based rather than problem based.

In the present study, we compare a metacognitive approach to SAT mathematics problem solving that involves teaching students to recognize problem types and then work backwards to determine what formulas, concepts and general problem solving strategies are needed to a concept/formula-based approach. We choose solving circle problems as a topic area. In the metacognitive strategies condition, instructional materials are prepared that cover the categories of circle problems and how to solve them using general problem-solving strategies such as part-whole or working backwards and specific circle-related formulae. In the concept/formula condition, instructional materials are taken from the College Board's own test preparation book (hence we call this the "College Board" condition) and are organized around concepts and formulas like finding the area of a circle. We hypothesize that students taught using metacognitive strategies will perform better than those who are taught using the College Board materials.

There is one other hypothesis that is investigated in the present paper. Prior research has shown that metacognitive math strategies can transfer across domains (cf., Everson et al., 1997). Therefore, while specific formula such as the area of a circle may not be applicable to other geometry problems, general problem-solving strategies such as part-whole reasoning and working backwards would be. Therefore, we would expect that metacognitive strategies would transfer from circle problems to other geometry problems, but concept/formula problems would not.Therefore, not only would we expect students learning metacognitive problem solvingprocesses to perform higher on circle problems than those learning College Board's concept/formula-based strategies, but the former students, by virtue of being able to transfer their metacognitive strategies to other problems, would also perform higher on other geometry problems as well. These hypotheses were tested with high school students who were preparing to take the SAT exams.

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Methods

Participants

Participants in the present study were 20 high school students recruited from local high schools in Fairfax and Loudoun Counties in Virginia. All were currently studying for their SAT tests and therefore were motivated to learn SAT techniques that would help boost their SAT scores.

Materials

There were two types of SAT materials that were used in the present study. The first type of SAT materials is the materials that taught students how to solve circle problems. There were two sets of these materials, one taken from the College Board's SAT preparation book and the others was the metacognitive strategies. In order to keep the two sets of materials as comparable as possible, each set was broken down by problem solving principles for which formulas were given and sample problems with solutions shown.

The College Board materials were broken down by circle concepts tested in the SAT: diameter, radius, arc, tangent to a circle, circumference and area. For each concept, relevant formulas were presented as well as problems embodying those concepts with the solutions worked out. The metacognitive strategies listed the formulas for things like circumference and area. However, rather than being organized by circle concepts as the College Board materials were, the metacognitive strategies materials were organized around problems types. By problem types, we mean the different categories of problem types that embody circle properties and formulas. These problem types were generated by reviewing released College Board SAT tests and categorizing the circle problems based on their themes.

For example, one problem type is to find a circumference of a circle given its area or the area of a circle given the circumference. Both involve working backwards from the information given and using the formula for area or circumference to find the radius and then using the radius to find the value being asked for. Another question type, related to the first, is to find a sector area given an arc length or vice versa. As with the previous type, each question version involves working backwards to find the radius and then plugging that into the arc or sector formula. Another problem type involved a smaller circle placed in a larger circle in which the diameter of the smaller circle has end points at the larger circle's radius and on the circle itself. These questions ask students to find areas/perimeters of one given information about the other. Again, students solve this type of problem by working backwards and noting that the radius of the larger circle is equal to the diameter of the smaller circle. The next type of question involves scale factors. Here, students are asked to find the area/circumference of a smaller or larger circle given a scale factor and the area/circumference of another circle. Another problem type involved

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part-whole problem solving where students are asked to find the area of irregular shapes that are part of larger shapes, which can be done by using the formula part + part = whole and finding the values of the whole shape and the other part(s) of the shape that are not asked for in the solution. A final question shape involved embedding other figures in circles or vice versa. Students are asked to find a value (like an area) of one shape using information given from the other shape. Typically, the other shape has overlapping parts or a key part of that shape provides a critical value needed to answer questions about the other shape (For example, if a circle is embedded in a square, the diameter of the circle is equal to a side length of the square. Therefore, knowing the area of the circle allows one to find its radius, meaning that doubling that radius and then squaring it will give the area of the square.)

The other set of materials in the study was the post-test. The post-test had a total of 30 questions on it. 20 of the questions were circle questions. These were taken from previous SAT tests and, therefore, were actual SAT math questions. The remaining 10 questions were geometry questions that involved shapes other than circles. The purpose of including these questions was to test whether learning metacognitive or College Board strategies for solving circle problems would transfer to non-circle geometry problems. These 10 problems were also taken from previous SAT tests.

Procedure

Participants were assigned to one of the two math strategy conditions (College Board strategies, metacognitive writing strategies). Participants were given a written document that outlined the strategies associated with the condition they were in. Once students received the written strategies, they were told to review them. After the strategies were given, students were given the practice packet described above. They were allowed to use the strategies packets while doing the practice problems. When they were through with the practice packets, they were given the 30-question post-test.

Results

Only the responses to the questions from the post-test were scored. There were 20 questions about circles and 10 about other geometric shapes for a total of 30 questions. Each participant's post-test was graded for the number of correct answers given. For the 20 questions about circles, the mean number of correctly answered questions for participants taught using the metacognitive strategy was 17.4 (translating roughly to a 705 math score or the 91st percentile), while the mean number of correctly answered questions for participants taught using the College Board strategy was 11.2 (translating roughly to a 555 math score or the 63rd percentile). This difference is statistically significant, $t = 4.71$, $df = 18$, $p < .001$.

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Additionally, the mean number of other geometry (non-circle) questions correctly answered by participants taught using the metacognitive strategy was 8.2(translating roughly to a 675 math score or the 88th percentile), while the mean number of other geometry questions correctly answered by participants taught using the College Board strategy was 4.2 (translating roughly to a 484 math score or the 37th percentile). This difference was also statistically significant, $t = 3.97$, $df = 18$, $p < .001$. Because the two subsections of the post-test showed statistically significant differences in participant performance based on teaching method, the overall difference between the mean post-test scores of 25.6 (metacognitive) and 15.4 (College Board) were also statistically significant, $t = 4.96$, $df = 18$, $p < .001$. These translated roughly to SAT math scores of 695 and 528 and percentiles of 90th and 53rd, respectively.

A further inspection of the individual performance scores suggests that not only did the metacognitive instruction led to higher overall performance than did the College Board instruction, but it also led to reduced variability in scores. Specifically, the scores for participants taught using the metacognitive strategies ranged from 22 to 28, while scores for those taught using the College Board strategies ranged from 6 to 25. This difference in variability was shown to be statistically significant, using a Levene's Test of Homogeneity, $F(1,18) = 9.86$, $p < .01$. The reduction in variability also held for circle questions, $F(1,18) = 8.79$, $p < .01$,where the score ranges were 14 to 19 for the metacognitive condition (translating to a range of 616 to 762 in scaled SAT math scores with corresponding percentiles of 78 and 96, respectively) and 6 to 17 for the College Board condition. It also held for non-circle questions, $F(1,18) = 14.47$, $p < .01$, where the score ranges were 6 to 9 for the metacognitive condition and 0 to 8 for the College Board condition. These results suggest that not only did using metacognitive strategies boost overall scores, but they also reduced variability, meaning that all students in the metacognitive condition performed reasonably well, whereas the performance of students in the College Board condition was all over the board.

A final research question posed by the present research is whether the metacognitive strategies would transfer to non-circle problems that embodied similar problem-solving principles. Above, we showed that participants in the metacognitive condition outperformed those in the College Board condition for non-circle problems. Another comparison that could be performed is to compare the proportion of circle vs. non-circle questions that were correctly answered. The closer these proportions are to each other, arguably the greater the transfer of instruction. The mean number of circle questions correctly answered in the metacognitive condition is 17.4 out of 20 or 87%. The mean number of non-circle questions correctly answered in the metacognitive condition is 8.2 out of 10 or 82%. These proportions are very close and the difference between them is not statistically significant, $z = .36$, ns. A similar analysis can be conducted for College Board condition where the percentages of circle and non-circle questions correctly answered are

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56% and 42%, respectively. While this difference is larger than the difference between problem types in the metacognitive condition, it is not statistically significant, $z = .74$, ns.

Discussion

The results suggest that using metacognitive math strategies results in higher scores in the SAT math section than using standard math strategies developed by the College Board itself. On average, participants using our metacognitive math strategies scored 17.4 out of 20 on the circle questions, equivalent to an SAT math score of roughly 705 or the 91st percentile, compared to 11.2 out of 20, equivalent to an SAT math score of roughly 555 or the 63rd percentile, for participants using the College Board strategies. Results also suggest that metacognitive strategies transferred to non-circle-based problems as participants taught metacognitive math strategies scored higher on non-circle problems than those taught the College Board strategies. Moreover, participants taught metacognitive strategies showed no difference in their problem-solving performance when given circle vs. non-circle problems.

There appeared to be a secondary benefit from using metacognitive strategies. Standardized tests such as the SAT are designed to produce something close to a normal distribution of test scores. Something like that appeared in the College Board strategy condition as scores on the circle questions ranged from six to 17 out of 20, with a mean of 11.2. On the other hand, the metacognitive strategies altered this distribution by producing a distribution of 14 to 19 correctly answered questions out of 20. The percentiles for this range are 78 to 96, respectively. This suggests that not only did the metacognitive strategies raise mean tests scores, they also eliminated average and below average scores, in this case, eliminating more than the bottom three fourths of the scores that most test takers get when taking the test. This suggests that metacognitive math strategies have the potential to make high performers out of virtually all students who use them.

Conclusion

As noted in the Introduction, people all over the world take high stakes assessments. In the present study, participants were taught the types of problems that math concepts appear in and generalized strategies (such as part-whole reasoning and working backwards) for solving them. The implications for this study are that it is not only important to teach basic math concepts, but it is also important for students to understand the categories/themes of problems they may encounter (whether on tests or in the real world) that use these concepts. This goes beyond the simple addition of word problems at the end of a lesson since students may not explicitly learn the pattern or theme behind the problem they are solving. Moreover, the implications of the present study are that problem solving often involves general strategies that supplement the specific formulas that are taught in specific units that are organized around topic. One apparent weakness of many textbooks is that lessons focus on topic-specific formula and do not include generic strategies that may be useful across diverse problem categories.

There is another benefit from including the metacognitive math strategies. Standardized testing often comes under fire for measuring test taking skills rather than the underlying content skills. When students are taught test-taking strategies in preparation for taking standardized tests, this feeds into this criticism. By showing that metacognitive math strategies lead to higher performance than do standard College Board test-taking strategies, it adds credibility that standardized testing measures math problem solving skills, even if also measuring test taking skills.

In the Introduction, we cited our previous work in using metacognitive reading and writing strategies to boost SAT reading and writing scores. The present research suggests that such strategies can work for SAT math questions as well. The present research should be extended to other subjects tested by standardized testing. This will extend the present findings, so that it can be determined whether metacognitive strategies can be used widely to improve test scores.

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