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# COGNITIVE STRUCTURE ANALYSIS: ASSESSING STUDENTS' KNOWLEDGE OF PRECALCULUS

Lebei Nobel Zhou and John Leddo

MyEdMaster, LLC, 205 Colleen Ct., Leesurg, Virginia, United States of America

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## ABSTRACT

Assessment has been a key part of education, playing the role of determining how much students have learned. Traditionally, assessments have focused on whether students give the correct answer to problems, implying that the number of correctly answered test items is a valid measure of how much students know. Unfortunately, the focus on correct answers has also resulted in neglecting the potential ability of assessments to provide diagnostic feedback to educators as to what concepts students have mastered and where the gaps in their knowledge are, thus potentially informing the day-to-day teaching process. The present paper describes an assessment technique called Cognitive Structure Analysis that is derived from John Leddo's integrated knowledge structure framework (Leddo et al., 1990) that combines several prominent knowledge representation frameworks in cognitive psychology. In a previous study (Leddo et al., 2022), CSA-based assessments of Algebra 1 knowledge correlated .966 with student problem-solving performance. The present study replicates the Leddo et al. (2022) findings on the subject of precalculus. Using a Google Form, students were queried on four types of knowledge considered the basis of mastery of precalculus concepts: factual, procedural, strategic, and rationale. From students' responses to these queries, measures of each type of knowledge and a combined knowledge score were created. Students were also given problems to solve. Correlations between each knowledge component score and problem-solving performance were high and the correlation between overall CSA-assessed knowledge and problem-solving performance was .80. Results suggest that CSA can be both easily implemented and highly diagnostic of student learning needs. Future research can investigate CSA's robustness across other subjects and whether incorporating CSA as part of day-to-day classroom instruction can lead to higher student achievement.

  

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## Introduction

Math requires students to actively shift their thinking, analyze problems from multiple perspectives, and learn by example, and is not something simply to be memorized. The emphasis in both education and assessments should be on skills and information management, as well as finding, analyzing, evaluating, and applying knowledge. Students must be guided by professionals to develop skills in a rich and complex environment where knowledge is constantly changing and expanding (Claro et al., 2018). In terms of equity, a guiding principle (Baker & O'Neil, 1995) is that assessment should promote open access to educational services for all students, regardless of socioeconomic class, religion, gender, ethnicity, or primary language (O'Neil Jr. & Brown, 1998). Therefore, the fundamental function of assessment is to present students' authentic academic performances. Previous studies indicated that open-ended and multiple-choice question formats have differential effects on metacognition, effort, and worry processes in student math achievement (O'Neil Jr. & Brown, 1998). Mathematics assessments among high school students are mainly focusing on the correctness of the answers provided. However, the rationale of education is to provide students with a mature system of interpretation of the contents in the first place, then the performance on exams and quizzes. Research indicated that teachers would be better able to recognize and examine common student misunderstandings of mathematical content and develop pedagogically sound practices if they focused on curriculum materials and the students' thought processes (Patel et al., 2012).

The reason why the participants of the present study are mainly taking math courses that are more advanced compared to precalculus is that previous research has shown that college students have wide disparities in their mathematics preparation, and many require remedial coursework before attempting college-level math; for instance, evidence shows that only 23% of 12th graders in the United States are proficient in mathematics (National Mathematics Advisory Panel, 2008). The NAEP long-term trend (LTT) reading and mathematics assessments have indicated that average math scores in a national sample of 13-year-old students fell by 9 points from 2020 to 2023(NCES, 2023). In addition, the average math score on the SAT shows a general downward trend from 2018-2022 (Nam, 2023), and on ACT, from 2018 to 2022, the percentage of students who passed the math section benchmark fell from 40% to 31% (Bushard, 2022). As a result, high school students may potentially have forgotten certain topics from lower-level math as well due to the lack of practice and consistent revision. Indeed, Leddo et al. (2021) found that students taking precalculus scored, on average, 84% on a 5th-grade math assessment, 65.5% on an Algebra 1 assessment, and 46.6% on an Algebra 2 assessment. Assessing students' ability to solve math problems at a lower level of difficulty than their current curriculum can help teachers identify underlying math proficiency issues and develop targeted practices to help students progress more effectively.

  

This study will replicate the procedure of a previous study completed in 2022 by Leddo et al. and will test the effects of the same assessment technique on examining high school student's understanding of precalculus concepts. The Cognitive Structure Analysis assessment method, based on John Leddo's integrated knowledge structure framework (Leddo et al., 1990), was used by Leddo et al. in the 2022 paper to assess students' understanding of algebra concepts. The cognitive structure analysis (CSA) assessment methodology aims to evaluate a student's fundamental knowledge concepts so that when a student makes a mistake, the cause of that mistake can be identified and remedied. This technique was developed in response to significant findings from years of cognitive psychology research, which demonstrated that individuals have different types of knowledge, each of which is organized and used differently in problem-solving. Because of the variations among individuals' approaches toward the problems, this framework makes the questions in four different forms to ensure the comprehensiveness of the assessment.

CSA incorporates four types of questions: semantic nets, which are used to organize factual information (Quillian, 1966); production rules, which are used to organize concrete procedures (Newell and Simon, 1972); scripts, which are general goal-based problem-solving strategies (Schank and Abelson, 1977; Schank, 1982); and mental models, which are used to explain the causal principle behind concepts (de Kleer and Brown, 1981).

Despite the prevalence of other types of assessments including multiple-choice tests, the practicality of this technique has been proven by previous studies in terms of problem-solving. In two of the previous studies, assessments created using the CSA methodology produced assessments of student learning that agreed with teachers' assessments 95%-97% of the time, which was statistically equivalent to teachers' assessments with each other (Leddo et al., 1998; Liang and Leddo, 2021). As a result, by using these assessment techniques to assess high school students' mathematical abilities, we hope to demonstrate their comprehensiveness and practicability, and thus provide a more well-rounded form of assessment that can enable teachers to recognize problems with individual students' learning.

## Method

### Participants

The participants were 16 high school students from various countries. The main population is Asian and Asian Americans. Students with a wide range of mathematical experience were recruited to ensure a range of knowledge levels among participants. At the very least, students were taking precalculus but may not have finished it. Some students at the upper end were already enrolled in Calculus AB or Calculus BC.

  

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**Materials**

A Google Forms survey was created for the study. The Google Form consisted of two parts: a CSA-based set of open-ended questions and a 20-question problem-solving test that was based on the concepts from the CSA portion of the assessment. The CSA-based questions covered facts, procedures, rationales, and strategies. The topic areas chosen were taken from precalculus. Topics and questions were chosen from Khan Academy and the precalculus textbook *Precalculus: Graphical, Numerical, Algebraic*. The specific subjects for the assignments were: trigonometry and complex numbers. This assessment is composed of four sections, fact-based questions, procedure-based questions, rationale-based questions, and strategy questions. The CSA-based questions are shown below.

**Fact-based questions:**

"What is the law of sines?"

"What can secant also be presented as?"

"What is the difference between sine30 in the first quadrant and sin30 in the fourth quadrant?"

"What is a unit circle?"

"What can tan(θ) also be presented as?"

"What can the imaginary unit "i" also be presented as? "

"What is the standard form for presenting the complex numbers?"

"What value is i^2 (i squared)?"

"Which axis is the real axis and which axis is the imaginary axis?"

"What is the polar form of the complex numbers?"

**Procedure-based questions:**

"How do you find sin(210°)?"

"How do you find the third length of a right triangle if you know the length of the adjacent and opposite sides?"

"How do you convert degrees to radians?"

"How to do find cosecant (π/4)?"

  

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"How do you add two complex numbers?"

"How do you multiply two complex numbers?"

"How do you conjugate complex numbers?"

"How do you find the distance complex numbers?"

**Rationale-based questions:**

"Why do you need to restrict the domain of the original function if you are going to find its inverse function?"

"Under what circumstance you will need to apply the law of cosine?"

"Why do you need to apply the Pythagorean Theorem under specific conditions?"

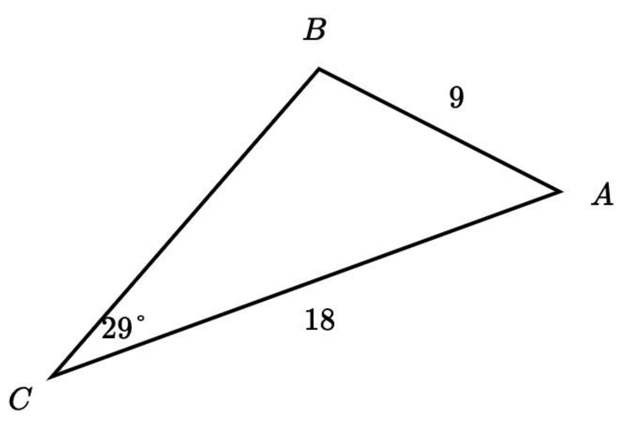
"Why do you need to conjugate the complex numbers if you are trying to divide complex numbers?"

"Why do you multiply complex numbers in polar form?"

"Why do we need the Fundamental Theorem of Algebra?"

**Strategy questions:**

The following figure shows $\triangle ABC$ with side lengths to the nearest tenth.




Find $m\angle B$.

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Provide your strategy for proving the equation below only instead of solving the problem.

$$\frac{2\tan x}{1+\tan^2 x} = \sin 2x$$

Provide your strategy for proving the equation below only instead of solving the problem.

**Find the distance $d$ between $z_1 = (-1+8i)$ and $z_2 = (-3-2i)$.**

*Express your answer in exact terms and simplify, if needed.*

$d =$

Provide your strategy for proving the equation below only instead of solving the problem.

$$z = -3 - 6i$$

**Find the angle $\theta$ (in *radians*) that $z$ makes in the complex plane.**

*Round your answer, if necessary, to the nearest thousandth. Express $\theta$ between $-\pi$ and $\pi$.*

$\theta =$

Following the four sections of knowledge assessment, participants were required to complete a post-test. The post-test consists of 20 questions, all of which were used to assess knowledge on the two main topics: trigonometry and complex numbers.

**Procedure**

The materials were administered in the form of a Google Forms survey. Participants were given links to the survey and asked to fill out the survey. They were given as much time as needed. Calculators were allowed, but no outside resources were allowed. Participants were supervised to prevent any use of outside resources.

**Results**

Students' written results were analyzed based on definitions provided in online math textbooks created by leading American publishers or educational learning sites such as Khan Academy. All students' results were analyzed using the same material and the same standard.

  

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The written section had 24 questions, the math strategy section had 4 questions, and the math problem-solving section had 20 questions, totaling 48 points for the entire questionnaire. Each written section question was worth one point. Instead of the word count, students' answers were evaluated based on whether they demonstrated a thorough understanding of the question when analyzing written questions. Students' answers that were similar to the official answers received full credit (1 point). Students' answers that were only partially related to the official answers received no credit. Students' answers that differed significantly from the official answers received no credit (0).

In the strategy section, students were given 4 mathematical problems and were asked to write down all the strategies needed to solve each problem. Students' results were analyzed based on whether they had fully demonstrated all the strategies needed to solve the problem. Students' responses to the strategy section were compared to answer keys that were composed after consulting reliable Mathematics websites such as Khan Academy. If students wrote all strategies needed to solve the problem, they received full credit (1 point). If students wrote parts of strategies needed to solve the problem, partial credit was given (0.5). If students did not write any strategy, no credit was given (0).

In the test section, students were given 20 mathematical problems and were asked to solve each question and provide the final answers. Questions were chosen from five topics: applications of the law of sine and the law of cosine, complex number evaluations, applications of complex numbers, trigonometric rules and evaluations, and application of trigonometrics. Students' answers were graded based on answer keys provided by math major undergraduate students from New York University. Students receive full credit (1 point) if they give the correct result. Students received 0 points if they did not get the correct answer.

In order to determine how well the INKS model could be used to model students' algebra knowledge and how well the CSA technique could be used to assess how much of that knowledge students have in a way that predicts their problem-solving performance, the knowledge component scores were correlated with problem-solving scores. The results of this analysis showed a correlation between total INKS knowledge as assessed by CSA and problem-solving performance of .80, df =14, $p < .001$.

The next step was to look at individual components of the CSA framework: factual, procedural, strategic, and rationale, and see how well they correlated with problem-solving performance. Factual or semantic knowledge correlated .649 with problem-solving performance, df = 14, $p <.01$. Procedural knowledge correlated .572 with problem-solving performance, df =14, $p =.02$, strategic knowledge correlated .537 with problem-solving performance, df = 14, $p =.03$, and rationale knowledge correlated .677 with problems-solving performance, df = 14, $p < .005$.

  

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**Discussion**

The current project's findings indicate the viability of using CSA to assess how well students have learned Pre-Calculus concepts. The correlations between the assessed individual INKS framework knowledge components and problem-solving performance were all high, and the overall correlation between the assessed overall INKS knowledge and problem-solving was .80.

The efficiency and effectiveness of the CSA technique are its main advantages. The current study used a Google Forms survey to implement CSA. This implies that it is easily scalable and can be implemented in the educational system with minimal disruption to existing practices and teacher training. This is a logical and promising next step in the current work.

Several questions remain unanswered in this study. The current CSA technique neither measures nor quantifies a student's proclivity to make careless errors on the final test score. This appears to be a manageable issue. In Liang and Leddo (2021), software was created to probe students' knowledge after they made mistakes in math problems. The software looked at the mistake that was made and then queried the student on the underlying INKS knowledge. If the student demonstrated mastery of the knowledge but still made a mistake, the software labeled the mistake as a careless one. This can be incorporated into the present framework in order to solve the problem.

This study focused on precalculus concepts in order to test the general feasibility of the INKS framework across different math subjects. The question is not only whether the INKS framework's basic predictive power holds, but also whether different types of knowledge must be included or different correlational strengths will emerge. Because precalculus is both factual and strategic in terms of problem-solving and applying previously learned knowledge, it is not surprising that factual and strategy-based knowledge were the most predictive of overall problem-solving among the four categories of INKS knowledge.

Trigonometric proofs are much more procedural in nature, so it may be the case that procedural knowledge will prove even more important for solving trigonometric proofs than it did for solving complex numbers equations. Similarly, while complex number requires students' capability to memorize factual knowledge and strategies, trigonometric application requires the capability of visualizing concepts as well as drawing parallels, which are skills that are not contained in the present CSA framework. CSA, and its theoretical basis INKS, may need to be expanded or even remodeled to incorporate this type of knowledge.

A fascinating question raised by the present research is whether students can self-assess using CSA. If this were possible, students could assess their own knowledge gaps and then implement corrective measures to fill them. One potential issue is that students are frequently unreliable in

  

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assessing their understanding of knowledge. Students will often self-deceive and make excuses for their mistakes, which will lead to discrepancies in their final assessment results. Leddo, Clark, and Clark (2021) found that middle schoolers who indicated that they understood algebra content they were taught answered correctly only two-thirds of questions based on that content. Moreover, when middle schoolers indicated that they did not understand a concept, they still correctly answered three-eighths of questions based on that content. These two results suggest that students in that study were not reliable in assessing their own knowledge of a subject matter they had been taught.

However, in the Leddo, Clark, and Clark (2021) study, students were not taught how to self-assess their knowledge; they simply relied on their subjective impression of whether or not they understood the content. CSA could serve as a reliable tool for helping students self-assess their knowledge. It may not be the case that students would be able to tell if their self-assessed knowledge is accurate (although they could fact-check it), but they may be able to use CSA to identify what gaps they have in their knowledge based on whether they can even answer the questions that comprise the CSA technique.

The most important research question that remains to be addressed is whether CSA, when integrated into daily classroom instruction, can boost student achievement. Here, teachers would use CSA, perhaps as part of the daily homework or in-class assignments, to assess how well students understand key concepts being taught. Any concepts that are shown to be deficient can be remediated. There is some preliminary data that suggests this may be the case. Leddo and Sak (1994) found that changes in knowledge as measured by CSA before and after instruction correlated .78 with changes in pre-test/post-test problem-solving performance after instruction was given based on the initially assessed needs. However, in this study, the assessment-instruction-assessment cycle occurred just once. Future research would implement the assessment and instruction cycle on a more continuous basis.

As evidenced by the current study's findings and the preceding discussion, CSA holds great promise as an assessment methodology for determining what students know and how knowledge gaps may impact performance, as well as part of a classroom instruction strategy designed to improve student performance. To address these issues, numerous research opportunities have been identified.

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