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COGNITIVE STRUCTURE ANALYSIS: ASSESSING ELEMENTARY SCHOOL STUDENTS IN MATH TO DETERMINE THE TYPES OF KNOWLEDGE THEY HAVE

Vishwa Bekkari and John Leddo

MyEdMaster, LLC

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ABSTRACT

Assessment has been a key part of education, playing the role of determining how much students have learned. Traditionally, assessments have focused on whether students give the correct answer to problems, implying that the number of correctly answered test items is a valid measure of how much students know. Unfortunately, the focus on correct answers has also resulted in neglecting the potential ability of assessments to provide diagnostic feedback to educators as to what concepts students have mastered and where the gaps in their knowledge are, thus potentially informing the day-to-day teaching process. The present paper describes an assessment technique called Cognitive Structure Analysis (CSA) that is derived from John Leddo's integrated knowledge structure framework (Leddo et al., 1990) that combines several prominent knowledge representation frameworks in cognitive psychology. Previous studies with middle and high school students revealed strong correlations between students' factual, procedural, strategic and rationale subject matter knowledge and their problem solving performance. Given that the Leddo et al. (1990) research showed that all four types of knowledge were present in highly experienced people but that novices tended to be characterized by factual and procedural knowledge, the question arises as to whether elementary school students would show significant correlations between only factual and procedural knowledge and problem solving performance on school subject matter due to their lack of academic experience. The present study investigated this question using fifth graders studying math. 20 fifth graders were taught beginning and advanced concepts in solving distance = rate * time problems. Afterwards, they were assessed on their concept knowledge using CSA and given practical problems to solve. Results showed significant correlations between factual and procedural knowledge as assessed by CSA and problem solving performance. Correlations between strategic and rationale knowledge were not statistically significant. These results suggest that elementary school students not only have less subject matter knowledge than their older counterparts, but that the

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type of knowledge they have is also qualitatively different. This finding may have implications for how to teach students in order to nurture advanced thinking skills.

Introduction

In the history of education, assessments have been used as a means of measuring the extent to which students have learned the content that they have been taught. In both classroom settings and in standardized testing, this content is operationally defined as the amount of correct answers a student gives on test questions. In the past, various frameworks have been utilized by teachers and educational organizations to test students' knowledge, typically categorized by whether students are required to select correct answers from a set of answer choices or required to construct their own answers to problems. While both categories of frameworks have their benefits, they do include drawbacks that affect their accuracy in assessing students' knowledge.

Multiple choice assessments require students to select and differentiate the correct answer choices from several distracting answer choices. They are widely used in standardized testing environments and classrooms due to their efficiency when it comes to grading (Chaoui, 2011). However, students often score higher on multiple choice tests than they do on constructive response tests as students can increase their scores through guessing (cf. Elbrink and Waits, 1970; O'Neil and Brown, 1997), which is often cited as a reason why multiple choice tests should not be used.

Constructive assessments require that students formulate their own answers to questions rather than choose from different answer choices as with multiple choice assessments. Researchers find, when investigating math problem solving, that students are more likely to use guessing strategies when given multiple choice tests but are more likely to reason mathematically when given constructive tests (Herman et al., 1994), thus increasing its validity in measuring students' actual learning (Frary, 1985).

These frameworks are based on Classical Test Theory, one of the major pillars of assessment methodology, which assumes that the total number of correctly-answered test items indicates the students' level of knowledge (cf., de Ayala, 2009). The challenge with the key assumption of Classical Test Theory, though, is that the assumption that correct answers indicate learning and vice versa may not be entirely true. A medical analogy works well here. Normally, if a person shows outward signs of illness, she/he is probably sick (although there could be non-medical reasons why a person can appear sick such as overexertion or lack of sleep). Similarly, a student who makes a lot of mistakes on a test probably has a lack of knowledge (unless, for example, they were distracted or sick during the test). However, a person can look healthy and still have an underlying illness. Similarly, a student may get correct answers on a test and have knowledge deficiencies. They could be regurgitating information or formulas that they do not truly understand or guessing correctly on multiple choice exams.

More importantly, the lack of correct answers does not inform the teacher as to what concepts need to be remediated. A doctor does not stop his/her diagnosis after observing symptoms. The doctor conducts additional tests to discover the cause of the symptoms, so that an appropriate remedy can be applied. Indeed, we would consider it medical malpractice for a doctor to treat only the symptoms and not the underlying causes of diseases. Similarly, an incorrect answer to a test question is a symptom that may indicate an underlying knowledge deficiency. We consider it to be educational malpractice to stop the assessment at that point without diagnosing the underlying knowledge deficiency that is causing that incorrect answer. Unless that cause is identified, how can the appropriate remedial instruction be given?

Therefore, our research indicates that there needs to be a new way to assess students' knowledge, one that takes advantage of testing all aspects of their knowledge. The current paper expands on a presented assessment methodology called Cognitive Structure Analysis (CSA) which was designed to assess the underlying knowledge a student has on certain subjects, so that when a student does make a mistake, the source of that mistake can be identified and refined. This methodology is based on decades of cognitive psychology research that have illustrated that people possess a variety of knowledge types, each of which are organized and used differently in problem solving. Due to the fact that there are different types of knowledge that people possess, our framework is an integration of several prominent and well-researched formalisms. These include: semantic nets, which organize factual information (Quillian, 1966); production rules, which organize concrete procedures (Newell and Simon, 1972); scripts, which are general goal-based problem solving strategies (Schank and Abelson, 1977; Schank, 1982); and mental models, which explain the causal principle behind concepts (de Kleer and Brown, 1981). Because our framework integrates these four knowledge types, it is called INKS for the Integrated Knowledge Structure.

We note that the National Council of Teachers of Mathematics (2000) has developed a taxonomy of strands necessary for students to be considered mathematically proficient that uses similar terminology: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning. In many ways, the strands of conceptual, procedural, and strategic do correspond to our own. The key difference is that the National Council of Teachers of Mathematics frames these strands in terms of desired skills/behavioral outcomes whereas the INKS framework conceptualizes these in terms of the specific knowledge needed to achieve those outcomes.

The INKS framework is based on research done by John Leddo (Leddo et al., 1990) which showed that true mastery in a topic or subject requires all four knowledge types. The framework

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also brings helpful implications for instruction. For example, in John Anderson's ACT-R framework, people initially learn factual/semantic knowledge that is later operationalized into procedures (Anderson, 1982). Research by Leddo takes this one step further showing that expert knowledge is organized around goals and plans (referred to in the literature as "scripts" – Schank and Abelson, 1977; Schank, 1982) and abstracted into causal principles (referred to in the literature as "mental models" – cf., de Kleer and Brown, 1981) that allow people to construct explanations and make predictions/innovations in novel situations.

In order to identify the root cause of the mistake, the query-based assessment framework, CSA, incorporates principles from the INKS knowledge representation framework. CSA is chosen because previous research describes a strong correlation between user knowledge — as assessed by CSA — and performance in practical problem solving. In one previous research project, we found that the using an automated multiple-choice CSA system to assess student learning produced measures of knowledge that correlated .88 with student problem solving performance and measures of change of knowledge as a result of instruction that correlated .78 with change in performance from pretest to post-test. Moreover, at-risk students taught based on the needs diagnosed using CSA performed at a mainstream level three grades higher than their own after a 25-hour tutoring program in science (Leddo and Sak, 1994). Leddo et al. (2022) extended these findings using CSA in an open-ended response format. In this study, students' algebraic knowledge was assessed and this knowledge assessment was then correlated with problem-solving performance. Students were given open-ended questions to assess their factual (semantic), strategic (script-based), procedural and rationale (mental model) concept knowledge of Algebra 1. The total INKS knowledge and individual component knowledge scores were correlated with the total number of correctly solved problems. Results showed correlations of .966 between problem solving and total INKS knowledge, .866 between problem solving and factual knowledge, .937 between problem solving and procedural knowledge, .819 between problem solving and strategic knowledge, and .788 between problem solving and rationale knowledge. Separate studies examined the effectiveness of CSA in assessing knowledge of precalculus (Zhou and Leddo, 2023) and the scientific method (Ahmad and Leddo, 2023). In two other projects, assessments produced using the CSA methodology produced assessments of student learning agreed with teachers' assessments approximately 95%-97% of the time, which was statistically equal to teachers' assessments with each other (Leddo et al., 1998, Liang and Leddo, 2020).

One of the things that the above studies that tested CSA as an assessment technique had in common was that they were done using middle and high school students as participants. Those studies showed that all four types of knowledge (factual, procedural, strategic and rationale) were predictive of problem solving performance. This is useful information in that it can help

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educators not only assess what knowledge students have but also students' level of intellectual development. As Leddo et al. (1990) showed, knowledge develops such that people develop factual and procedural knowledge first and strategic and rationale knowledge as they acquire greater levels of expertise. Accordingly, if CSA were to be used as an assessment tool for elementary school students, it might be revealed that those students lack strategic and rationale knowledge and that only factual and procedural knowledge would be predictive of problem solving performance. This hypothesis was tested with middle school students in the subject of math.

Method

Participants

The experiment contained 20 5th grade students from Liberty Elementary in Virginia. All students were in the same math class and the experiment took place in that class during regular school hours.

Materials

There were two assessment instruments that were created. The first was a knowledge assessment that asked students open-ended questions about the problem types D=RT or Dfinal = Dinitial + RT. The knowledge assessment contained questions about the factual, procedural, strategic, and rational knowledge needed to solve these problems as described in the Introduction section above. The second assessment instrument consisted of 11 word problems of which five were D=RT problems and six were Dfinal = Dinitial + RT problems.

Procedure

Participants were first given the pre-test to ensure that they did not already know how to solve math problems of the form D = RT or Dfinal = Dinitial + RT. Students were then taught, in a classroom setting, how to solve these types of problems. Upon completion of the instruction, they were given a written post-test that consisted of both the CSA-based knowledge assessment and the 11 problem-solving questions.

Results

As discussed in the method section, there were two types of assessments that were provided to Participants. The first tested their concept knowledge and the assessment items were broken down into categories of factual, procedural, strategic and rationale knowledge. The second assessment type tested Participants' ability to solve problems using the concepts they were taught. In the second assessment, there were two types of problems: easier problems that could

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be solved using the basic d = rt formula (Problem Type 1) and more difficult problems that could be solved using the formula final distance = initial distance + rt (Problem Type 2).

By using two types of problems, a harder and an easier problem type, the present study was able to examine the robustness of the CSA technique within the same population and same general topic area on problems of different difficulty. In order to confirm our assumption that Problem Type 2 was more difficult than Problem Type 1, we calculated the percentage of correct answers for each problem type given by each student. For Problem Type 1, the mean percentage of correct answers given by Participants was 82; for Problem Type 2, it was 46. Using a paired t-test, it was determined that Problem Type 2 was indeed a harder problem class as the percentage of correct answers was significantly lower than that of Problem Type 1, $t = 5.59$, $df = 19$, $p < .0001$.

For the conceptual knowledge questions, Participants' answers were graded per question as either 0 or 1 where 1 point was awarded if Participants' answers were substantially correct and 0 points were awarded if their answers were not substantially correct. For each Participant, a score was calculated for each of the knowledge types - factual, procedural, strategic, and rationale - and for the total number of correct answers given, which represented the sum of the correct answers to each of the four knowledge types. For the problem solving questions, again, 1 point was awarded for each correct answer and 0 points were awarded for a wrong answer. The total scores for both types of problems were calculated for each Participant.

As was done in the Leddo et al. (2022) study, once the assessment scores were calculated, correlations were performed between the concept and problem-solving knowledge scores. Overall concept knowledge scores and the Problem Type 1 problem solving scores correlated .59, $df = 18$, $p < .01$. Overall concept knowledge scores and the Problem Type 2 problem solving scores correlated .53, $df = 18$, $p < .05$.

The correlations between the factual knowledge scores and the Problem Types 1 and 2 scores were .60, $df = 18$, $p < .01$ and .58, $df = 18$, $p < .01$, respectively. The correlations between the procedural knowledge scores and the Problem Types 1 and 2 scores were .47, $df = 18$, $p < .05$ and .44, $df = 18$, $p = .05$, respectively. The correlations between the strategy knowledge scores and the Problem Types 1 and 2 scores were .29, $df = 18$, ns and .11, $df = 18$, ns, respectively. The correlations between the rationale knowledge scores and the Problem Types 1 and 2 scores were .06, $df = 18$, ns and .12, $df = 18$, ns, respectively.

Discussion

The present study showed that the CSA assessment technique could predict problem solving performance in elementary school students. This was tested across two different types of

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problems, and the results showed a similar pattern for each type, suggesting a consistency in the methodology. Specifically, overall knowledge, as measured by CSA, correlated significantly with how well Participants could solve problems. Moreover, both factual and procedural knowledge individually correlated significantly with how well Participants could solve problems. On the other hand, strategic and rationale knowledge showed no correlation with problem solving performance. This is consistent with the experimental hypothesis.

Given that a major purpose of the present study was to replicate the Leddo et al. (2022) findings for algebra with a younger population studying pre-algebra topics, it is worth comparing the results of the present study with those of the Leddo et al. (2022) study. In doing so, two major differences stand out. First, in general, the correlations found in the present study are much lower than those found in the Leddo et al (2022) study. Second, only two individual knowledge types (factual and procedural) showed statistically significant correlations with problem solving performance, whereas in the Leddo et al. (2022) study, all four knowledge types showed statistically significant correlations with problem solving performance.

The fact that the correlations in the present study were lower than in the Leddo et al. (2022) study may be explainable by the relative ages of the participants in the two studies. The participants of the present study were fifth graders, while the participants in the Leddo et al. (2022) study were middle and high school students. Generally, as one progresses through a subject, one acquires more knowledge about that subject that can be used to solve problems in that subject. Therefore, it is logical to assume that those with more knowledge of a subject area will rely on that knowledge when solving problems in that subject and show a greater correlation between knowledge and performance, while those with less knowledge of a subject area may rely on other types of knowledge or heuristics when solving problems and, therefore, show a smaller correlation between knowledge and performance. The question of what other types of knowledge students use to solve problems when their subject matter knowledge base is undeveloped is an interesting research question that deserves more attention.

The second major difference between the results of the present study and those of the Leddo et al. (2022) study is that in the present study only factual and procedural knowledge had statistically significant correlations with overall problem solving performance whereas in the Leddo et al. (2022) study, all four knowledge types (factual, procedural, strategic and rationale) showed statistically significant correlations with problem solving. This finding can be explained using the knowledge progression model articulated by Leddo et al. (1990). Leddo et al. (1990)'s knowledge progression model represents an extension of the ACT model proposed by Anderson (1980). Anderson's ACT model argues that people initially learn factual knowledge ("knowing that") and later proceduralize that knowledge in order to solve problems ("knowing how"). Leddo et al. (1990) studied people with greater levels of expertise than those reported in the

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Anderson studies and found even higher levels of abstraction. Leddo et al. (1990) found that people not only proceduralize knowledge but also create knowledge hierarchies of those procedures around goals and plans (script-based knowledge) and later abstract that goal and plan knowledge even further into rationale knowledge (mental models).

Given that the fifth graders in the present study were far less experienced in math than the middle and high schoolers in the Leddo et al. (2022) study, it is reasonable to conclude that the fifth graders had a lower capacity for strategic and rationale-based processing than middle and high schoolers did. This would explain why strategic and rationale knowledge strongly predicted problem solving performance in middle and high schoolers but not in elementary schoolers.

The findings of the present study raise important questions that merit further investigation. First, given that CSA is effective in predicting math problem solving performance in fifth graders, is it effective in predicting problem solving performance in other disciplines, particularly those that might be less factual or procedural in nature? For example, essay writing, a skill fifth graders learn in school, requires strategic knowledge as students organize their essay and rationale knowledge as they logically defend the assertions they make. Given that strategic and rationale knowledge showed no significant correlation with problem solving for math, but may not be as necessary for elementary-level math, the question arises whether these types of knowledge will correlate with essay writing where they are more necessary.

Second, generally, the purpose of assessment is to measure what students have learned in order to support teaching. Often, assessments of problem-solving performance are used to identify which types of problems students have trouble with. Corrective instruction often centers on reviewing how to solve those types of problems or showing solutions to the problems that were missed and then giving more practice. The CSA paradigm suggests a new model of assessing the knowledge components a student has and lacks and then remediating at the knowledge level. The next logical step in this research progression is to investigate whether remediating students based on identified knowledge deficiencies will lead to improved problem solving performance compared to a more traditional approach.

Third, the results of the present study are consistent with the original framework proposed by Leddo et al. (1990) regarding the progression of knowledge development. Collectively, the CSA studies show that elementary school students' knowledge centers on facts and procedures, while older students have these types of knowledge plus strategies and rationales. This raises the question of whether we can teach the more advanced knowledge types to younger students and, if so, how would we do that? This is an important area of research since it may help lay the foundation for developing society's future experts.

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