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An In-Depth Analysis of The Synergy/ Confluence Between the Concept of Mathematics with Sound and Modern Technology

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ABSTRACT

Mathematics and sound have an extremely close relation with each other. This is seen when Pythagoras discovered his theorem and used it extensively as a musician. All acoustics and modern forms of music and sound take the help of mathematical concepts in executing the best sound effects. Mathematics is an essential part of all sound systems whether they are in the form of instruments or auditoriums.

Research Question: The paper would attempt to analyze the relationship between sound and mathematics. What are the concepts of mathematics that are used in sound engineering? How far can the enhancement concerning mathematical expertise help in better sound effects? How closely are these two concepts related? Who is the type of people who would like to take advantage of their synergy? These and similar questions will be attempted to be answered in the course of research.

Introduction

Mathematics is the science of all sciences and an art of all arts. It has been a playback pioneer in scientific advancements of modern civilization and is a subject that constantly changes. In the 21st century, the Western view of mathematics is the abstract science of space, shape, change, number, structure, and quantity. Pythagoras, Plato, and Aristotle were the 3 very clever academics and influential figures. The relationship between mathematics and sound extends to many elements of both. Studying one could lead to success in the other. Though the two are entirely different fields, at some point there is a strong correlation between them and they tend to overlap. It is quite common for mathematicians to be good musicians. Generally, a musician to better himself tends to turn towards the study of mathematics. The two sciences may seem to be completely unrelated to each other, but they share a profound symbiotic relationship that stretches back centuries. The age-old stereotype of mathematicians that they are "left-brain

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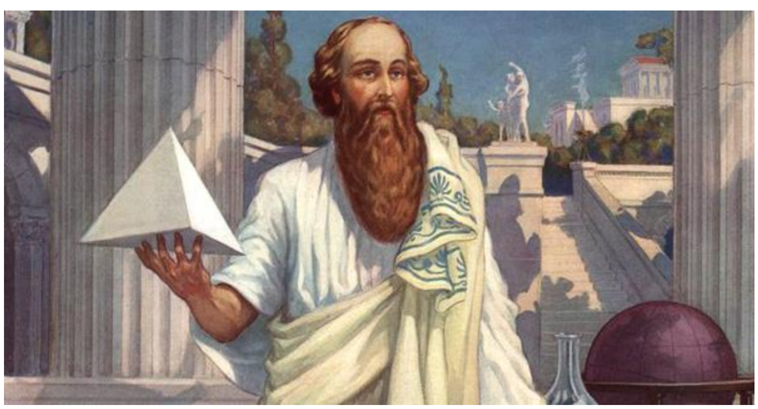
scholars disinterested or incapable of soulful expressions" is not entirely true. Professor Cymra Haskell at the USC Dornsife College of Letters, Arts & Sciences who plays the fiddle said "When I write down a proof of a result in mathematics, it feels like a puzzle coming together. There can be intense pleasure in that, similar to the pleasure I feel when I listen to a beautiful piece of music or gaze at a beautiful painting or go for a walk on the mountain, on the beach, or in the woods." This clearly indicates that there is immense pleasure and happiness in mathematics, as in the other so-called art installations.

Symbolic mathematics with equations, proofs, and theorems was discovered about two thousand five hundred years ago. Calculus was not developed in the 17th century, negative numbers, modern abstract algebra where symbols like X, Y & Z. This was used only hundred and fifty years ago.

Pythagoras theorem:

The first concrete argument for a fundamental between mathematics and sound was made by an earlier philosopher and mathematician Pythagoras (569-475 BC). He, once having ascertained that the pitch of notes depends on rapidity of vibration, and also that the planets move at different rates of motion. He concluded that the sound made *by their motion* should also vary according to *their different rates of motion*.

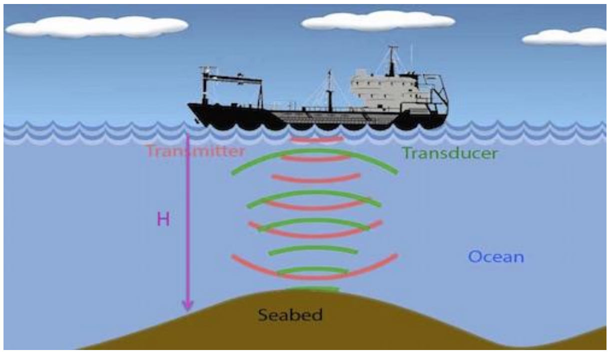
Figure 1: Pythagoras (Greek mathematician and philosopher)



Source: https://www.themarginalian.org/2018/05/23/pythagoras-olympic-games/

The Pythagoras theorem that is used to check whether a given triangle is right angle or not is used by aerospace scientists and metrologists, besides musicians to find the range and sound source using the Pythagoras theorem. It is also used by oceanographers to determine the speed of sound in water.

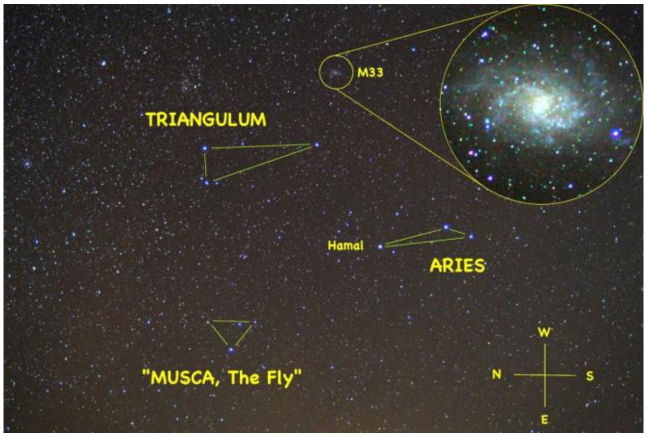
Figure 2: Pythagoras used by oceanographers



Source: https://plus.maths.org/content/saving-whales-using-pythagoras

This theorem was credited with many mathematical and scientific discoveries including the Pythagorean theorem, Pythagorean tuning, the 5 regular solids, the theory of proportions, the sphericity of the earth, and the identity of the morning and the evening stars as the planet, Venus.

Figure 3: Astronomers using Pythagoras theorem



Source: https://www.steamboatpilot.com/news/celestial-new-the-triangle-the-ram-and-the-fly/

The other real-life uses of this theorem are in construction and architecture. It is also used to survey the steepness of slopes of mountains and hills.

Pythagoras and Ptolemy are the first to have investigated a relationship between math and sound. Pythagoras noticed a pleasant relationship between sounds produced by similar strings of equal thickness and tension when the ratio of the length was 2:1 (Octaves), 3:4 (Perfects fourths), and 3:2 (Perfect fifths). He quoted "There is geometry in the humming of the strings, there is music in the spacing of the spheres." He examined hammers that were harmonious with each other

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sharing a relationship in their respective weights, they were made of simple fractions such as one-half or one-quarter. He, thus rationalized that a 1:2 ratio produces an 'octave', the same note with a higher pitch.

It was this discovery that was used to understand the concept of 'sound'. 'Sound' is a vibration that travels as an acoustic wave through a medium such as air, gas, liquid or solid. As sound is not visible to the human eye, it can be depicted through a wave equation in one dimension: $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

An example of a one-dimensional medium is the guitar string. The energy travels in the direction of the unplugged string while the wave-like motion is perpendicular. Sound waves that travel through the atmosphere are an example of compression waves. The compression occurs due to high and low-pressure regions.

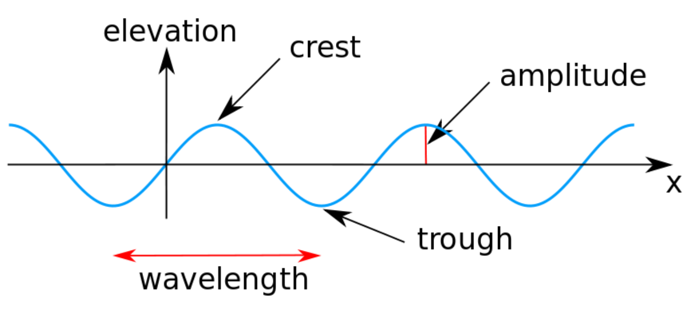
The wave equation in spherical coordinates: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u}{\partial r}\right) = \frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2}$

In open air, waves travel in all three dimensions. If they are confined to the interior space of a hollow tube then they would travel in one single dimension. The smaller the tube, the closer to reality.

If the sound is confined to moving between two very large flat sheets lying side by side, then it is known as movement between two dimensions. This phenomenon is true if air is confined between two concentric tubes: One lying within the other.

The amplitude of a sound wave can be defined as the maximum displacement of vibrating particles of the medium from their mean position when sound is produced.

Figure 4: Characteristics of a wave



Source: https://en.wikipedia.org/wiki/Wave_height

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Sound Engineering:

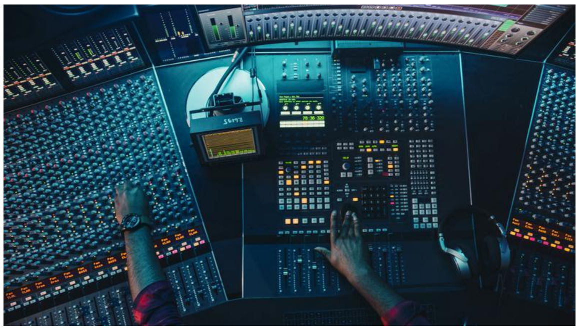
This involves the scientific aesthetic and technological aspects of manipulating recording and reproducing sound. It applies electronic, digital, acoustic and electrical principles to the recording and production of music voice, and sound.

Audio engineers are responsible for capturing, mixing, or reproducing sound using electronic audio equipment. The field is broad since it's applied to music, television, film, and other media channels.

Mathematics is used to compare and contrast the amplitude, envelope and spectrum of sound. Music and math are connected through various principles and structures like:

1. Rhythm and time signatures: Music is based on rhythm which involves the division of time in two regular intervals. The time signatures in music are 4/4 or 3/4, indicating the rhythmic structure of the piece. The division of time is closely connected with fractions and ratios.
2. Harmony and intervals: This is described in terms of intervals such as the octave (2:1 ratio), perfect fifth (3:2 ratio), and perfect fourth (4:3 ratio). These correspond to mathematical relationships between frequency.
3. Frequency and pitch: The pitch is determined by the frequency of a sound wave.
4. Mathematical structures in music composition: Techniques like fugues, cannons and palindrome structures are mathematical concepts applied to music composition.
5. Music scales involve patterns of interval that can be described mathematically.
6. In modern digital music, there is a high dependence on mathematics. Signal processing algorithms such as Fourier transformation are used to analyze and manipulate audio signals to create effects like equalization, reverb, and compression.

Figure 5: Sound Engineering

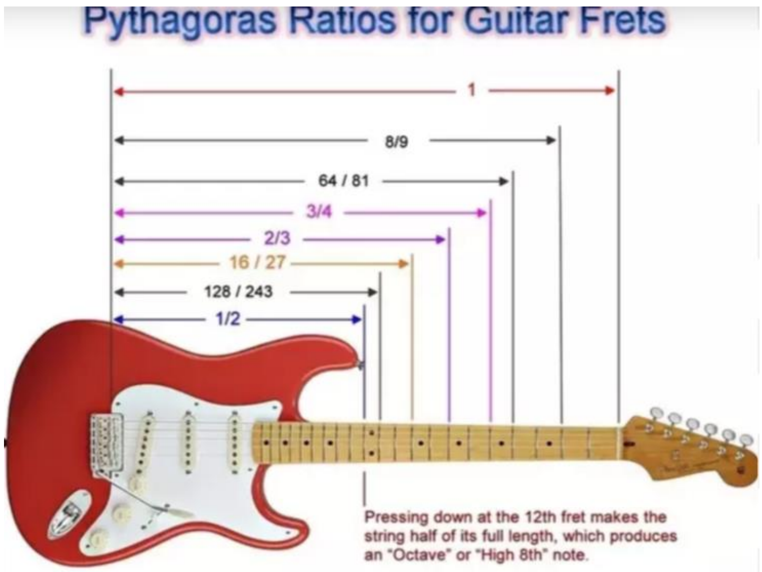


Source: https://www.backstage.com/magazine/article/audio-producer-equipment-guide-74460/

The synergy between music and math:

The sound wave is modeled on the wave function s(x,t)=x_{max}sin(kx+$\omega t + \varphi$)

Figure 6: Pythagoras Ratios for Guitar Frets



Source: https://www.wikipedia.org/

Music theory does not have any axiomatic foundation in modern mathematics but the basis of sound is described mathematically through acoustics. The concept of ratio is used between the pitches. Operations such as transposition and inversion that are called isometries are used to preserve the intervals between tones in the sets. Some theorists have used abstract algebra to analyze music/sound. The concept that is used is the *abelian group*. The transformational theory emphasizes transformation between musical objects. The theory of regular temperaments is developed with the help of sophisticated mathematics. For example, each regular temperament is associated with *Grassmannian*.

Other concepts that are used are the *golden ratio* and the *Fibonacci sequence*

The mathematician and musicologist Guerino Mazzola used the *category theory* in (topos theory). This includes using topology as a basis for a theory of rhythm and motives and differential geometry as a basis for *musical phrasing, tempo,* and *intonation*.

Sound is measured by intensity which is also known as sound pressure or sound power. It is measured in units called decibels (dB).

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The role of Fourier Analysis:

Fourier analysis is a pivotal mathematical tool used in the study of sound.

This method is named after the French mathematician, Jean-Baptiste Joseph Fourier.

Fourier transforms are used to decompose waveforms into a set of discrete frequencies and to reconstitute them. This analysis is used as a univariate method for simplifying data of a modeling. It could also be used as a multivariate technique for data analysis and evaluates the relationship of sets of data of different perspectives.

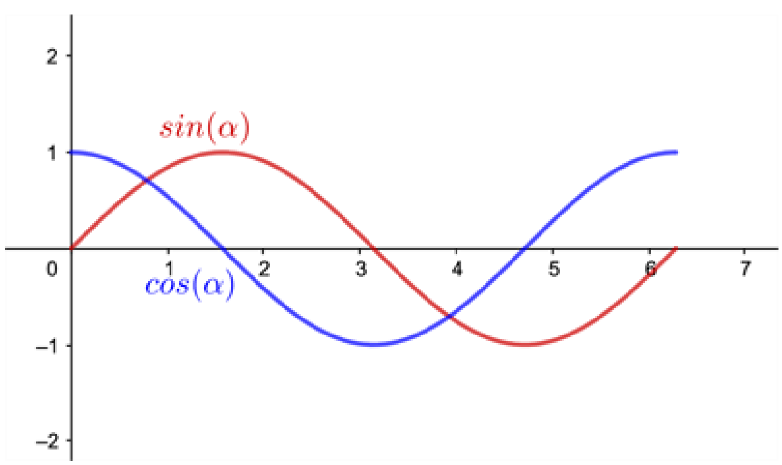
The applications of this theorem are used extensively in physics, partial differential equations, number theory, combinatorics, signal processing, probability theory, statistics, option pricing, cryptography, numerical analysis, acoustics, optics, etc.

Fourier's theorem: It states that any repetitive waveform can be represented as a collection of sine and cosine waves of the proper amplitude and frequency.

It is a periodic signal in terms of cosine and sine waves and if we assume $0 \leq x \leq L$ periodicity, then the Fourier's theorem states that $f(x)$ can be written as:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi x}{L} + b_n \sin \frac{2n\pi x}{L} \right)$$

Figure 7: sine and cosine waves



Source: https://plus.maths.org/content/why-sine-and-cosine-make-waves

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The main advantage of this analysis is that very little information is lost from the signal during transformation. The real-life applications besides analysing differential equations are in spectroscopy, NMR, MRI, quantum mechanics, nuclear signs, earthquakes and sound analysis.

Musical Acoustics:

Acoustics is the science of sound, and its connection to mathematics is particularly evident in music. Musical notes correspond to specific frequencies and the relations between these notes can be depicted using mathematical ratios.

Western music, a system of 12 notes arranged in increasing pitch, known as octaves is central. The tuning of these notes is not random; It is mathematically determined.

Mathematics also plays a role in other areas of classical music. Ken Alexander, a professor of mathematics at USC Dornsife, played the flute for 30 years and he believed that there's a strong connection between the skills needed to play an instrument and those essential to solving math problems.

Additionally, a study found that older adults with little prior musical experience improved their memory after taking four months of keyboard harmonica lessons.

Figure 8: Musical acoustics

Source: https://www.tunedly.com/blog/mathematics-and-music-are-related!-tunedly-online-music-recording-studio.html

The designs of spaces like concert halls, recording studios, etc depend highly on acoustic principles to ensure that there is optimal sound quality.

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Applications of Math in Modern Music:

Mathematics is applied in the tone system, the frequency of two notes that have an octave difference bear the ratio 1:2. If the frequency of *Shadja* (The tonic note) in the middle octave is equal to n vibrations per second, then the frequency of the higher shadja would be 2n. That of the next would be 4n and so on. Thus, the frequency relationship of the octaves proceeds in a geometrical progression.

Further, in the concept of the sXI degrees of speed (Shatkalas), one finds a regular progression. In this case, the length of the noth gets progressively reduced unit time in the first degree of speed to 1/2, 1/4, 1/8, 1/16, 1/32. This means that one note is sung to unit time in the first speed, two notes in the second speed, four notes in the third speed etc.

The arithmetic progression 1, 2, 3, 4, 5… is seen in the frequency relationship of *upper partial tones* (Upper octave of the prime tones). The harmonics are heard when a stretched string is sounded.

Using the concept of twelve notes, the 72 melakata or scales have been evolved through permutation. Other scales in raga have also been arrived by permutations or omission or addition of the twelve notes.

The swara graphs show the contours of a raga while swarasthana and struthistana graphs indicate the frequency of the notes in the ragas.

The sapta tala gives rise to the 35 talas because of the five types of laghu.

The Pallavi of a ragam-tanam-pallavi is highly mathematical and requires the musician to sing the same line in different degrees of speed, gati etc. It is mounted with a lot of mathematical structures in the kalpanaswara. Singing under the kalpanswara pattern requires a reasonable amount of mathematical knowledge. Musical patterns like the yati (Eg- gopucha, damaru, mridanga etc have specific geometrical patterns).

The attempt to structure and communicate new ways of composing and hearing music has led to musical application of set theory, abstract algebra and number theory. Some composers have incorporated the golden ratio and Fibonacci numbers into their work. By applying simple operations such as transposition and inversion, one can discover the depth of structures in music. Operations such as these are called isometries because the preserve the intervals between the tones in a set.

Music and Abstract Algebra: The chromatic scale is a musical scale that is a sequence of 12 pitches. The interval from one pitch to one next to it is called semi-tone.

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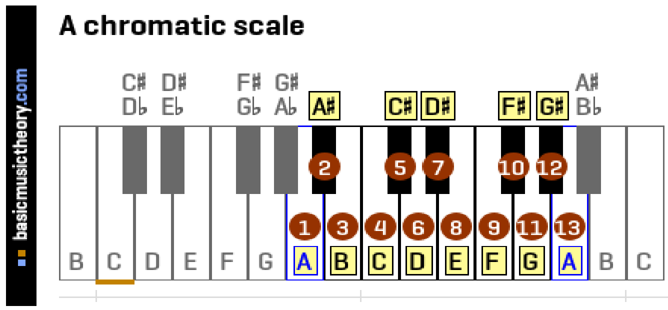
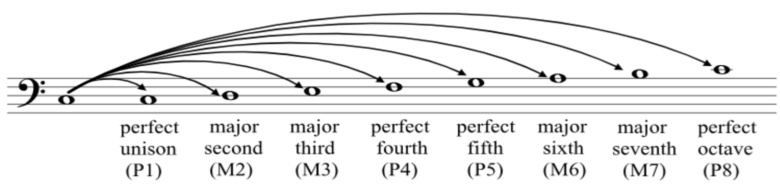
Figure 9: Chromatic Scale on Piano



Source: https://www.basicmusictheory.com/a-chromatic-scale

Each interval is defined by the number of semitones it contains and the set of these intervals forms a binary operation of addition.

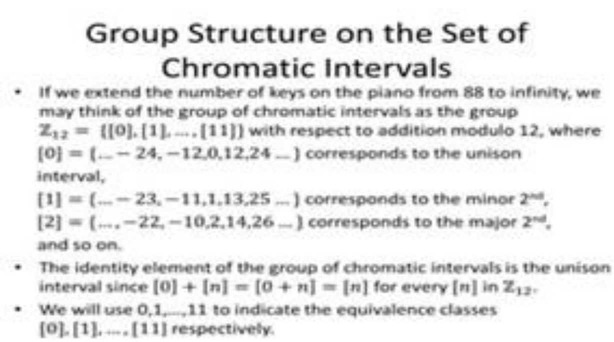
Figure 10: Introduction to intervals



Source: https://musictheory.pugetsound.edu/mt21c/IntervalsIntroduction.html

If the number of keys on a piano is extended from 88 to infinity, then a group of chromatic intervals are formed.

Figure 11: Group structure on a set of chromatic intervals



Source: https://www.slideshare.net/slideshow/an-application-of-abstract-algebra-to-music-theory/12612165

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Abstract algebra is also applied to music theory. Example- Tonnetz. Essentially, all music theory is abstract. A high 'E' or a low 'E' relies on an algebraic idea. Neoriemannian terminology also uses abstract algebra. Bach, when he wrote his music, used calculus. Music theory is a field within mathematics. Group theory, a branch of pure mathematics came from abstract algebra. The musical notes that are used are the following: Musical flat b, music sharp #.

Music and Number theory: The pitch classes in an equally tempered octave form an abelian group with 12 elements. Each piece of music has a time signature which gives it rhythmic information. All notes are music have a numerical value.

Sounds are produced by vibrations, which are variations in air pressure that travel from the source of sound to the human ear, where they are processed and sent to the brain. Different variations in air pressure cause different shaped waves. The moment sound touches the membrane of the ear, it can be depicted by a differential equation: $\frac{d^2y}{dt^2}$ = -ky, where t is the time, y is the distance to that point on the membrane from its equilibrium solution. The solution to this differential equation indicates the basic building block that is used in acoustics.

Music is based on 'beats' to produce a rhythm. This is done via prime factorization. For ex- If the question is, what is the combination of triplets over rhythmically inclined quarter notes? The answer would be to subdivide the beat into triplets and quarter notes. So that the combination of the two could be heard.

How many beats would be required to subdivide the measure?

The answer is the least common multiple (LCM) of 3 & 4 which is equal to 12.

Golden Ratio: This occurs in music in two forms:

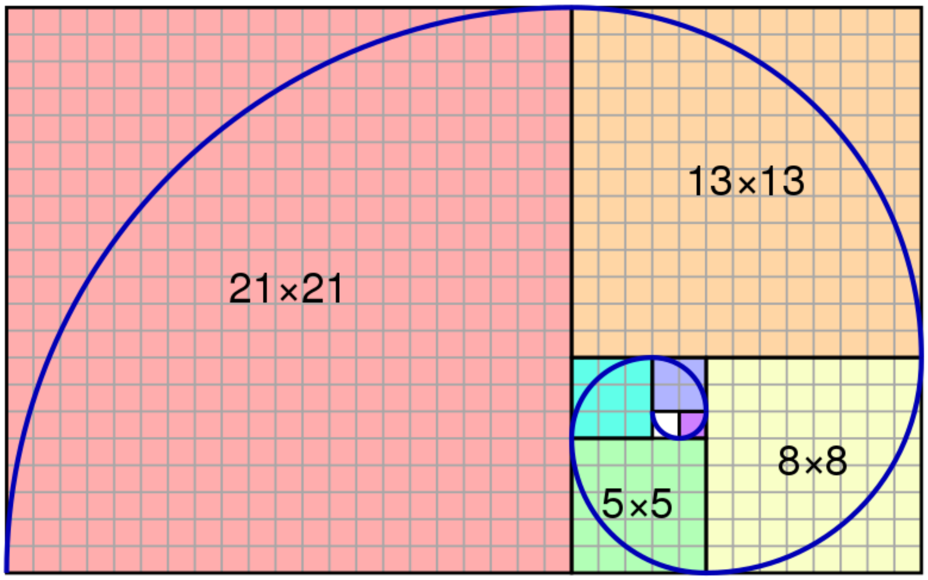
1. 2 notes or their frequencies can relate to each other in the proportion of the golden ratio.
2. The composition of a piece can consist of parts whose lengths can be in the proportion of the golden ratio.

The golden ratio is the interval between the carrier and the modulator, such that the resulting timbre is an inharmonic cloud of golden ratio-related partials. The ratio names**φ** (phi) of height to width to length of a room to achieve optimal sound. This is found in the 2/1, 3/2, and 5/3 ratios and is present in two beats, three beats, and five beats measure.

Mozart based many of his works on the golden ratio, specially when he was playing on the piano. He arranged his piano sonata so that the number of bars in the development and recapitulation divided by the number of bars in the exposition would equal 1.618.

Beethoven golden ratio in the Fifth Symphony was 600:372

Figure 12: Golden Ratio



Source: https://elementor.com/blog/golden-ratio/

Importance of electromagnetic waves

Sound waves are not electromagnetic in nature. They are mechanical waves that require a medium such as air, water, or a solid to travel but they can be converted into electrical signals which is a form of electromagnetic radiation. This is achieved using transducers. Sound waves must travel through matter by bumping molecules into each other and they cannot travel through a vacuum-like space. Electromagnetic waves do not need molecules to travel.

The earliest experiments involved using microphones and transducers to convert acoustic vibration into electrical signals. These signals could then be amplified and projected through speakers, opening up new possibilities for musicians. One of the first instruments was the electric guitar.

Electromagnetism affecting sound

The motor effect is used inside the headphones which contain small loudspeakers. In these devices, variations in an electric current cause variations in the magnetic field produced by an electromagnet. This further causes a cone to move which creates pressure variation in the air and forms sound waves.

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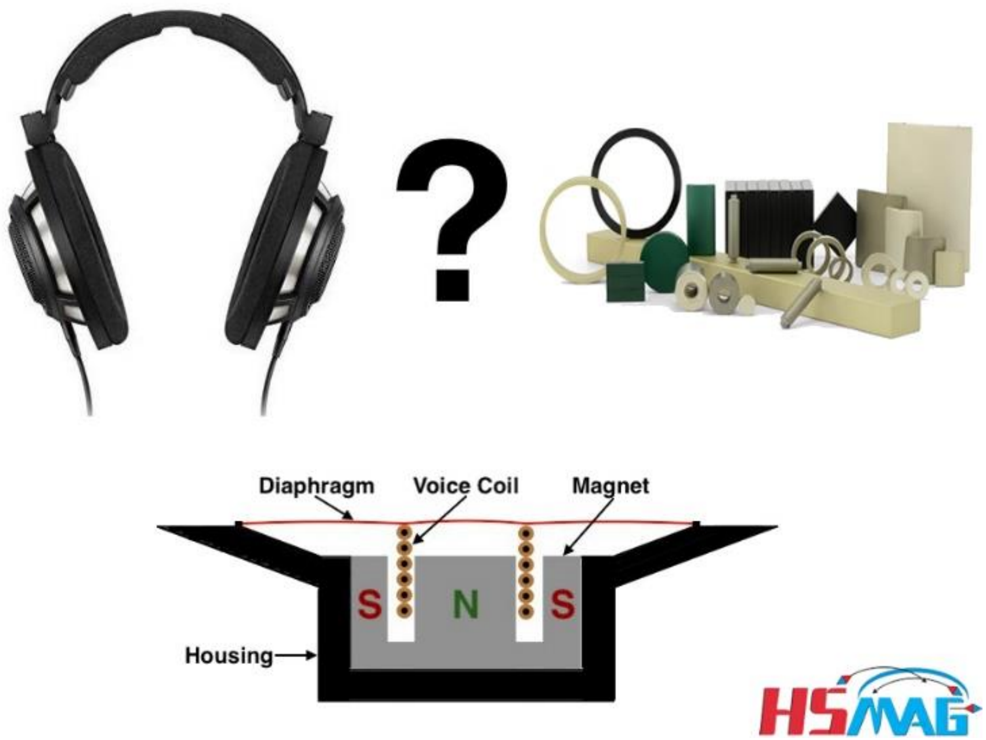
Figure 13: Electromagnetic waves in headphones



Source: https://www.hsmagnets.com/

Humans are incapable of hearing electromagnetic waves. The current frequency for tuning musical instruments is 440 Hz (20 to 200 Hz is known as the bass frequency, 4000 to 20,000 is the treble frequency, and between 200 and 4000 is the mid-range frequency.)

Relationship between principles of mathematics and rarefaction and compressions of air molecules

Compression is a region in the longitudinal wave where the particles are the closest together and rarefaction is where the particles in the longitudinal wave are the farthest apart. This is used in sound waves. It indicates the pressure on the particles. Pressure does not have direction; it is the force that has the direction. The particles in a compressed state have very little displacement i.e. they do not have enough space to move freely and are thus at high pressure. In the case of rarefaction, the movement of particles is high and they are at low pressure.

Particles from compression are attracted towards rarefaction. This process continues till the energy is exhausted.

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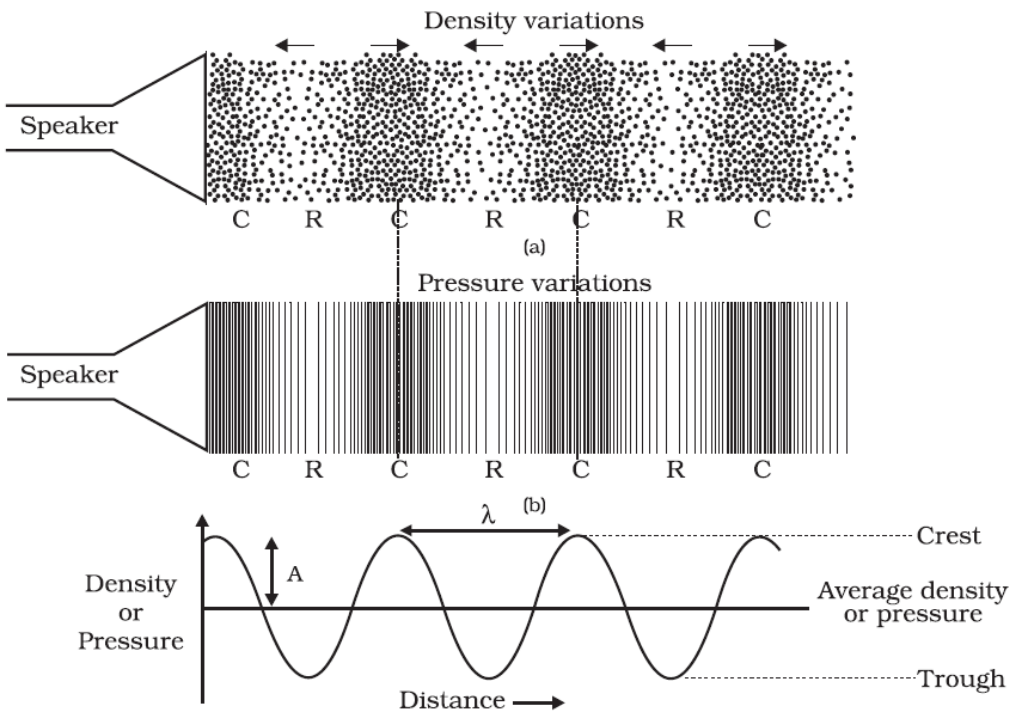
Figure 14: Rarefaction and compression in sound waves



Source: https://byjus.com/

Vector spaces

The intervals between musical notes can be regarded as vectors in a vector space. Intervals in the diatonic scale have natural 1-, 2-, and 3- dimensional vector representations, and there are also natural mappings from 2 to 1, 3 to 1, and 3 to 2-dimension.

The harmonic heptagon provides a compact visualization of all the consonant relationships between notes in the diatonic scale.

Vectors are mathematical objects with magnitude and direction. They can be formulated in terms of components.

Finite dimensional vector space V is defined as follows:

It is of 'n' dimensions where n is finite.

2 vectors from the same space V are equal if and only if all their corresponding components are equal.

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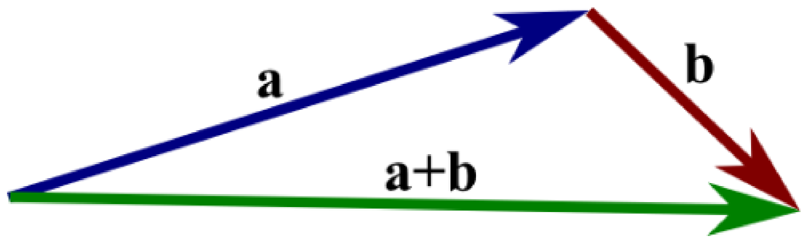
Vectors from a vector space V can be added together by adding their corresponding components.

Figure 15: Vector



Source: https://mathinsight.org/vector_introduction

Discoveries being developed in this field

Besides Pythagoras and Fourier series, Gallelio's theory in a rhythmic canon problem in the field of minimalistic music as well as a neat word theory theorem discovered in a construction originating in combinations of mystical octaves and fifths in Plato's *Timaeus*

The relationship between mathematics and music encompasses Category Theory, Topology, Graph Theory, Homology, Differential Calculus, Abstract Algebra and Linear Algebra.

It has been argued that several advances in the so-called hard science were made possible by musical queries.

Musical innovations opened new valleys of thought that helped develop new concepts, eventually unraveling original results.

Conclusion

There is an extremely close relationship between mathematics and music. In fact, a large number of musicians are mathematicians. The counting rhythm scales intervals, patterns, symbols, harmonies, time signatures, overtones, tones, pitch, are all seemingly musical concepts but each one of them has a mathematical theory behind it. The father of this twin concept was Pythagoras who is an extremely famous mathematician and musician. Even the German composer Bach produced so scrupulously structured music that it was aptly compared to mathematics. Iannis Xanakis, a French Greek musician, apparently incorporated mathematical modelling such as stochastic processes in his compositions. Recently, Gallelio's theory in a rhythmic cannon problem, and the neat word theory has been discovered in Plato's *Timaeus.*

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There is a continuous endeavour to discover and incorporate newer and newer synergies between the two. There seems to be an extremely close connection between the seemingly so-called diverse areas of music and mathematics.

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