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WEALTH AND INCOME DISTRIBUTION: A REVIEW TOWARDS NEW TRENDS

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ABSTRACT

The increasing inequality observed in recent years between the income of 1% of the population with respect to the rest, a new interest arose in the study of the distribution of wealth and income, questioning the effectiveness of traditional economic models even provoking economists to consider other approaches to study this problem. In present paper a revision of some of these approaches is made, from the statistical part, going over Pareto´s work, and the later works of Gibrat, Piketty, among others, and how complexity sciences have contributed through the econophysics and Agents-Based Models, from a Bottom-Up and Top-Down perspective, until the emergence of complex networks. Getting with it a contribution to find a more robust view of the problem.

Keywords: Wealth and income Distribution, Power Law, Economic Complex Systems, Econophysics, Agent-Based Models, Complex Networks

1. INTRODUCTION

With the 2008-2009 financial crisis, economic models applied to development and wealth generation were questioned by its efficacy. Besides the crisis, the growing incomes and wealth distribution inequality around the world [1] pointed out the necessity of, as Bouchaud mentioned, a scientific Revolution to construct new paradigms in economic theories [2]. About this growing inequality, former U.S. President Obama considered the worst threat of our times (U.S. Former President Barack Obama's speech in 2013 “The defining challenge of our Time”). Doyney Farmer and Duncan Foley stated in [3] that the economical mainstream is getting unviable to predict crisis as the 2008 mentioned one. In November 2010, European Central Bank (ECB) President Jean-Claude Trichet also mentioned the “serious limitations” of existing dynamic stochastic general-equilibrium and econometric models not only to predict a crisis but to prevent or control any once it happens [4]. Fontana made some consideration to shift neoclassical Samuelsonian economy paradigm to a complex theory approach [5]. “Economics can do better, it’s time to move on” (Beinhocker, 2006, p. 23) [6]. Since then, what have been these

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"revolutionary methods" that could aid classical mainstream economic models? Although some of these methods are not new, but its implementation to explain and model "stylized facts", as power tails, provide new insights to develop solutions to deal with these emergent phenomena. Through this work we will give a brief review of these methods from complex systems to complex networks. This survey starts in 1895 when the economist Vilfredo Pareto highlighted for the first time that wealth of a nation was distributed by a simple "power law", today known as "Pareto Law", finding that this law was present even in different countries and through different historical ages, as Abul-Magd [7] showed that wealth distribution in ancient Egypt behaved as a power law as well. This discovery, today considered as an Universal Law, triggered the formal use of statistics to economic issues. In 1931 Gibrat found that Pareto Law fitted only to the high-income tails [8], between 1% and 5%, to the rest of the curve a log-normal distribution is approximated. In 1953 Champernowne modeled income distribution with a markovian process [9], [10] but until computer capacity was improved this model was able to be verified with available empirical data. Since the outstanding Adam Smith's work about the wealth of nations ("*An Inquiry into the Nature and Causes of the Wealth of Nations",* 1776) the interest about wealth generation and its distribution has been considered the normative economic issue 'par excellence' [11], being Economy discipline the most interested in explaining the observed empirical results. In 2014 one of the more powerful works about wealth and income distribution is the one developed by Thomas Piketty [12] which marks a shift towards the study of historical information about many societies and nations, obtaining a relation between per capita income rate (income growth) and rate of return on capital ***(r > g),*** this means that capital income increases as part of overall income, explaining with this famous relation part of observed results. Other approaches have been emerged from other science fields to explain Pareto Law [13], Jones [14] made an analysis of Pareto's Law and Piketty's results. In this essay we do a survey of these approaches, form statistics to the arising of physics branches dedicated to probability analysis (statistical physics) (Stanley, 1996), which has been specialized on developing phase transition models and correlation analysis, study of many particles physics or scaling terms, now this physics field is also studying financial markets or economic systems and their emergent behaviors [15], which led to the creation of econophysics as a subject field contributing to unify economic areas usually separated in their analysis, macro and micro economy (Jaansen, 2006). Besides, with thermodynamics and many particles statistical physics approaches, which are closely related, it is possible to construct models through complex systems theories [16], a lately fast growing knowledge field involved in solving social and economics phenomena such as wealth and income distribution problems [17]. Complex systems are characterized by its high connectivity between systems agents (among other properties) which are easily represented by complex networks structures [18]; [19], enabling to observe the power law in the scale free networks and to model economic exchanges in small-world complex

[20]. In this document we will refer to studies based mainly on empirical quantitative information, so we do not enter into a more ideological or narrative analysis about possible causes or consequences (World Bank Group, "World Development Indicators", 2017). Atkinson & Bourguignon [21] after their first publication of a series of Handbooks released in 2000, they published the second volume of this Handbook of income distribution, as an effort to continue with trends on this topic, reviewing the changes occurred over the next fifteen years on income distribution and inequality. Stiglitz mentioned in his book "The Price of inequality" [22], which is a remarkable reference to get insight in recent researches of empirical data collection and in some narrative explanations of income distribution inequality. These inequality consequences have attracted the attention of top international financial organizations (World Bank, IMF, OECD, Davos Meeting, etc.), concerning them on solutions to the problem. Present essay is organized as follows: in section 2 some statistical models dealing with wealth and income distribution are showed. In section 3 a brief description of complex economic systems related to wealth and income distribution is shown, while section 4 gives an overview of econophysics applied to economic systems and how it is addressed from the three proposals, statistical physics, bottom-up and top-down agent-based models [23]. In section 5 some of the main works about wealth and income distribution studied through complex networks approach are mentioned (Figure 1).

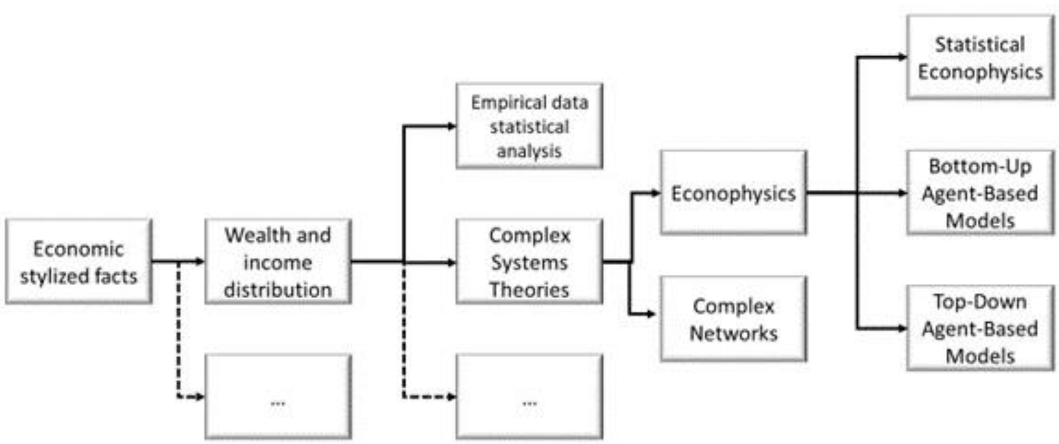

Fig. 1: wealth and Income distribution approaches reviewed in present paper.

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2. EMPIRICAL STATISTIC AND STOCHASTIC MODELS

In this section, a historical review of wealth distribution models will be made from a mathematical approach based on the results found by the sociologist, economist and civil engineer civil Vilfredo Pareto (1896), who first studied income per person through the information collected from the tax offices in a stable economy from a statistical approach fitting his observations to the distribution known as power law [21]. In his pioneer work he also noticed that wealth distribution obeyed this Power Law even for different countries and in different historical periods [7], [24]. A research done by Lee et.al [25] demonstrated that the growth of complex organizations obeys a power law distribution as well. Since then, both economists and sociologists [26] and recently econophysicists, have been given the task of modeling the corresponding form of distribution from economic data, as well as to build theoretical models capable of reproducing the mentioned distribution. Being one of the pillars of the study of the economy, the distribution of wealth and income has led to great questions such as what its form is, its causes, its consequences. Pareto analyzed that the relationship between the logarithm of income per person x and the logarithm of N_x, the total number of income recipients greater than $x(x > x_o)$, linearly decayed as:

$$\log N_x = A - \mu \log x$$

where,

$N_x = e^A x^{-\mu}$

with A, α > 0. Normalizing $N_x = N$ it si obtained what has been termed as a Universal Law, "Pareto's Law",

$$P_>(x) \sim (\frac{x_0}{x})^\mu \qquad \text{... (1)}$$

Where $P_>(x)$ corresponds to the probability of finding an agent with an income greater than x, and μ is known as the Pareto Index, which corresponds to an exponent with value around 1 for individual wealth and for companies' sizes as well. For values $\mu \approx 1.5$ it is called Pareto´s Strong Law [10]. Currently it is more common to use the density function, $P(x)$,

$$P(x) \sim x^{-(1+\mu)} \qquad \text{... (2);}$$

For large values of *x (Figure 2)*.

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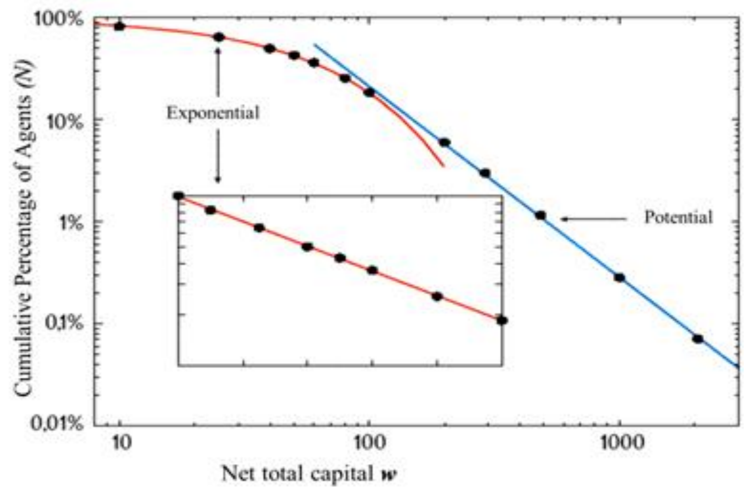

Fig. 2: In the abscissa axis Wealth (w) is represented in a logarithmic scale; ordinate axis presents percentage population (N) with a wealth greater than a certain amount in a logarithmic scale. For an 95% of the population the curve behaves as an exponential distribution or logNx=A-µlogx. Inside frame represents same distribution in a logarithmic scale comparing to linear scale. For the rest 5% of higher incomes a power-law is showed (equation (2)).

Until 1931, this distribution was considered for the entire income range but it was the work done by Gibrat [8], who identified that for the middle income region the Pareto distribution was not adjusted by proposing that for theses ranges, based on available empirical data, income was distributed as a *log-normal* density function:

$$P(x) \sim \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left(-\frac{log^2\left(\frac{x}{x_0}\right)}{2\sigma^2}\right) \quad \dots (3)$$

Where the mean $\langle \log(x) \rangle$ corresponds to income lohgarithm $\langle \log(x_0) \rangle$ whose variance is:

$$\sigma^2 = \langle [\log(x) - \log(x_0)]^2 \rangle$$

of which Gibrat defined the index:

$$\beta = \frac{1}{\sqrt{2\sigma^2}}$$

known as the Gibrat index and used as an index of inequality. From this work it was recognized that the distribution of income and wealth presented different regions for the high income ranges

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and for the rest of the population, this is that around 1% to 5% corresponding to the upper-tail of the distribution it behaves as a Pareto distribution and for the rest have been adjusted to the aforementioned log-normal distribution, and already with the work related to econophysics, which are described below, an adjustment to a Gamma distribution is also made [27].

Two works stood out for their applied method, Champernowne [9] and Mandelbrot [10], who used a stochastic process to describe the income in such a way that the income for a current period is described through the Markovian transition matrix through the following model:

$$X_s(t+1) = X_r(t)T(r,s|t)$$

Where the number of income receivers is defined by the vector $X_s(t)$ in the income range ***r*** and in the period ***t***. The evolution of the process clearly depends on the shape of the stochastic matrix $T(r,s|t)$. The work carried out by Mandelbrot, also develops a Markov process, adjusting the results to a Pareto-Levy distribution. The results obtained by Champerowne and Mandelbrot have been reference for investigations resumed 40 years later and thanks to the improvement in the computing capacity and the available information, interest in the subject [16]. To resume this idea, it is worth mentioning the contribution of Simon Kuznets,(he pointed the necessity to develop a reliable data source for both income and wealth (savings) [28] [29], managing to collect information regarding this inequality in income and wealth in different countries, achieving projects such as the Luxembourg Income Study (LIS) and its complement the LuxembourgWealth Study, years later forming a more robust repository the World Income Inequality Database (WIID) along with the World Top Income Database(WTID) have been led from the work of Piketty, Atkinson y Saez [30], as well as surveys conducted by Gallup WorldPoll, in addition to sources of free access from the World Bank (PovcalNet) [31].

Although the reliability of the data was not entirely accurate for high levels of wealth as opposed to the measurement of income since information sources for income are usually obtained from the tax offices and the respective national accounts, the problem is that wealth is not always declarable [32]. It is from the efforts of Piketty, and later Atkinson, to collect and organize historical information about the distribution of income in 26 different countries to be able to carry out a subsequent analysis in the outstanding work of Piketty, "Capital in the 21st Century" [12], with which research was unleashed on the subject by incorporating more information from more countries, verifying the empirical behavior in the distribution of income and wealth, which for the high income range obeys a Pareto Law, and for the rest of the middle-income population, it adjusts to a log- normal in different researches like:

United States (Figure 3) [33]; Japan [34]; Rumania (Figure 4) [35], United Kingdom and Germany [36], [37]; Italy [38], India [39], Norway [40], Mexico (Figure 5) [41], among others.

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These works have focused on the statistical part of the empirical information, the distribution modeling is addressed in the next section, but it is worth mentioning besides the works of Gibrat [8], Champernowne [9], Kalecki [42] and also other outstanding works have allowed to model the Pareto Law or power [43], including the work done by Piketty [12] whose contribution highlights the effect on the tax on capital not only reduces the accumulation of wealth, but also the structure of the distribution of wealth in the long term, from that conviction is built the famous model that relates the per capita income rate and the rate of return ***(r-g)***, that has caused controversy, because in previous works they are not related to the distribution of income or wealth but in their work together with their collaborators they manage to identify that exponential growth as a law of powers [14], so ubiquitous relationship in economics and finance. What has been discussed in this section about theoretical models based on empirical econometric data does not refer to the activity of heterogeneous income-receiving agents or generators of wealth, new mathematical tools were required beyond the classical mathematical structure of the economy to be able to handle the Stylized Facts which are observed in the distributions observed in the economy and finances [44],specifically Pareto's law. An analysis about the statistical models applied in the distribution of wealth and income can be reviewed in [45].

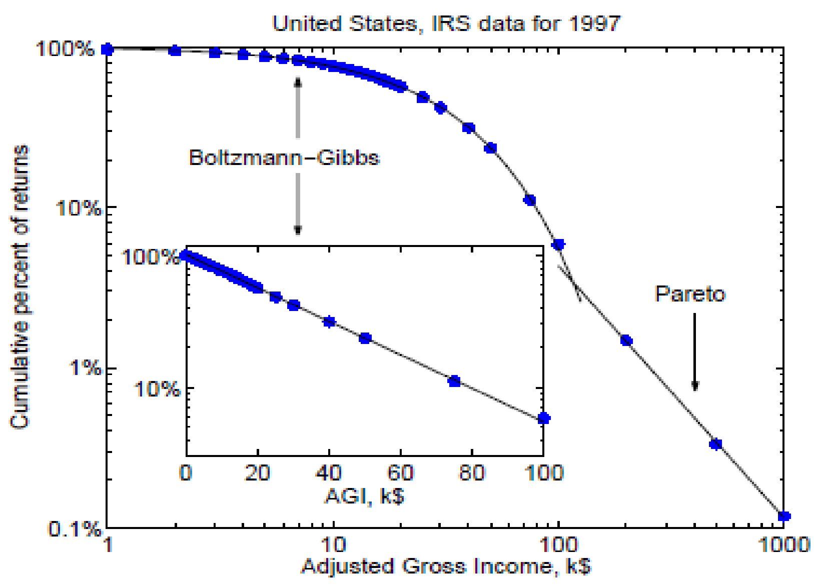

Fig. 3: Cumulative probability distribution for the 1997 US tax office data about income per person in the log-log scale in the main table, and in the internal table, it is reported in a log-linear scale. Reproduced from: [33]

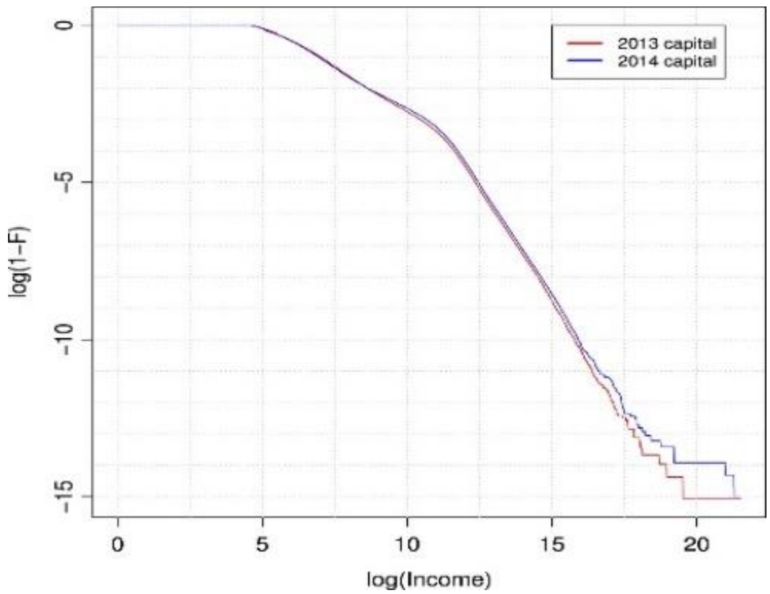

Fig. 4: Log-log plot for Rumania´s capital income distribution for the years 2013 and 2014. Reproduced from Oancea et al. 2018

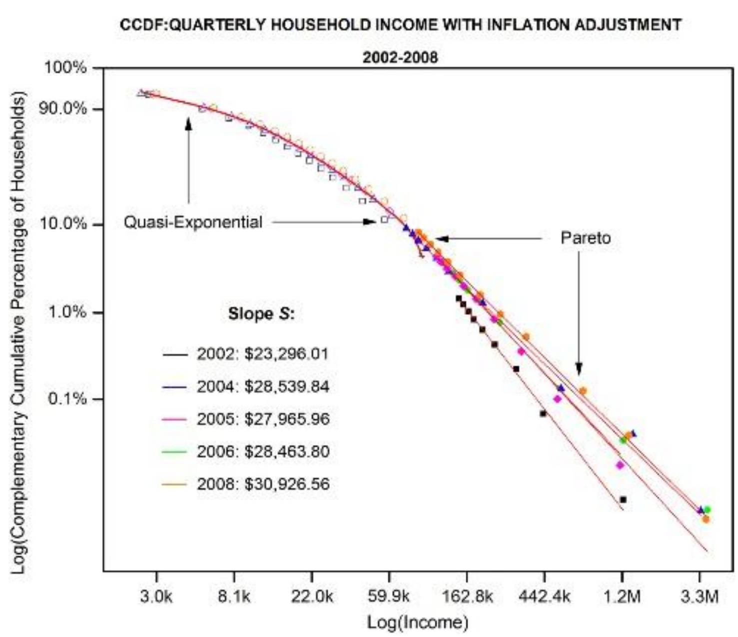

Fig. 5: Cumulative distribution function of income per household with adjustments to inflation for the years 2002 to 2008 in Mexico on a log-log scale. It highlights the power function behavior for the high-income range and exponential for the rest. Reproduced from: Soriano-Hernández et. al, 2017.

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3. COMPLEX ADAPTIVE SYSTEMS

In this paper we avoid to mention the basic theoretical framework presented in the economic neoclassical mainstream with its solutions approximate to a General Equilibrium, in order to omit any arbitrary assumptions or guesswork common in this framework, but an analysis of models built up on real economic evidence as the known stylized facts [46], i.e. the power law formation, is reviewed. On the contrary to mainstream approach where economic phenomena are inferred out from rational agents, whose predetermined behavior drive them to a certain equilibrium, real economic stylized facts as power law tails, emerge from a continuous adaptive out of equilibrium interactions of an enormous number of heterogeneous agents with high uncertainty decision making environment.

For a description of an economic system, it should be consider its stochastic dynamics, correlations between variables, the infinity number of agents as well as the great diversity among them, the self-auto organization, auto-similarity, unpredictable time series which arise to chaotic process, scale factors observable in power laws, so the methods applied in econophysics are adequate to treat complex systems present in physics, chemistry, biology and economics [47]. The empirical results reviewed from the stochastic approach, as the agent-based models, show a behavior of income distribution and wealth as power law suggesting that the endogenous mechanisms that generate the distribution are complex [48]. Also it is observed that the dynamics in financial transactions, (investments, credits, financial derivatives, crisis in the stock markets, etc.) defined as complex processes are more common in the upper part of income reason why they obey more to a power law of Pareto [41]. The importance of the distribution of wealth and income, considered as the emergence of the interaction of many individuals, lies in the social interest of inequality. From the perspective of the economy, knowledge in the statistical properties of the origin of income and wealth distribution are essential in the study of macroeconomic activities, as well as in business cycles, and since Adam Smith, it has been a central point in macroeconomics research. These concepts are mainly associated to complex systems behavior. With recent economic crisis fundamental flaws of neoclassical economic theory are highlighted mainly its inability to describe economic phenomena. Since Walras adopted for economic models the classical mechanics framework of explaining the behavior of the compounds of agents in a “reductionism” approach, i.e. aggregate dynamics can be determined by the sum of its single components behavior. This assumption is true if no interaction between agents is taking place and if agents are full rational belonging the complete information. In presence of interaction in fact, the aggregate is different from the sum of its components. The only interaction of agents possible in the mainstream economic model is an indirect one, through the price system only. But with the development of Quantum revolution the reductionism hypothesis was dismayed in the sense that particles properties are described only

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through analyzing the aggregate, i.e., the whole determines the behavior of the parts. Analogously to quantum physics, economic agents do not exist if they are not connected to each other, arising an instability network structure where a small perturbance can cause different outputs. This last description is one of the statements of complex systems theory, developed through interaction of many different theories psychologists, anthropologists, sociologists, historians, physicists, biologists, mathematicians, computer scientists and others across the social and physical sciences. Other characteristics which Economic systems present are:

- many heterogeneous agents, not only rational as in a Neoclassical economic framework,
- feedback loops, as in income and consumption relation, i.e., when income is big, consumption increases producing an increase in income.
- Economic systems are self-organized as Adam Smith's postulation of invisible hand states.
- In present description of wealth and income distribution, there is a tendency on the analysis of power law observed with different approaches, considering income distribution, which is a macro-economic behavior, as an emergent phenomenon coming from interactions of agents in the system. Power law tails as an emergent phenomenon present in many economic time series, different to the Gaussian distributions associated to the Walrasian Neoclassical Theory (the Walrasian equilibrium model was devised by the nineteenth-century French economist Leon Walras (1834-1910)).
- As mentioned lines above, an economic system is an open system which is constantly perturbed by its interaction with its environment (international commerce, natural resources, political structures, etc.), avoiding an equilibrium state.
- Furthermore, present state of an economic systems is the result of its past interactions that implies that an economic system is not static, it is evolving and in constant adaptation. Consideration to new technologies which generates new markets, as the NASDAQ one, is an example of this evolution and adaptation.

For a detailed description and better insight on economic complex systems refer to [6], [49], [50]. Notable contributions to development of complexity economics emerged from Santa Fe Institute in the 1990's highlighting the work developed by Brian Arthur on increasing return [51]. Notorious differences are shown between neoclassical economic framework and complex system approach for income distribution discussion, [52], other interesting discussion is done by John Foster considering mainstream economics approach as a nonrealistic modeling paradigm, practical, but not enough efficient [53] resulting complex systems theory more realistic than General Economic Equilibrium (or the partial) models and as Bouchaud and Beinhocker mentioned, economy needs a new shift to a new paradigm, able to explain emergent stylized facts.

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> 'We have to rethink the way in which economic policy is conceived and enacted… […] Far from advancing toward a precise analytical model capable of being used for forecasting, and thus of guiding economic policy, the nature and ambitions of economic policy would have to change' [54].

Vast literature has been developed about economic complexity, Aruka and Kirman [50] edited a work where some important facts of economic complexity were stand out. Another remarkable work by Commendatore [55] studied three important structures in economic complex systems, the spatial macro, meso and micro levels involved in the different elements interacting in the corresponding systems. With the aid of high computational resources available so far two important methodologies were built to model economic complex systems. One approach had to do with the emergence of macro-patterns coming from spontaneous agent's behavior, named as agent-based models. The other approach deals with macro-statistical regularities. The first one has a deductive approach by means of micro-based behaviors used in economic frameworks, and the second one refers to an inductive methodology through observation of statistical patterns [56], [57]. A more detailed discussion of this last approach is shown in next section.

4. ECONOPHYSICS

4.1 Statistical Econophysics

An approximation to the study of wealth distribution is under the address of econophysics, an emerging branch of physics established formally around 1996 when referring to the knowledge of physics applied to the economics issues (Mantenga & Stanley, 2000), whose term "econophysics" was first introduced by Eugene Stanley in 1995 at the conference Dynamics of Complex Systems in Kolkata India, (Chakrabarti, 2005) but was appeared first in [58]. Although a first approximation of physics to economic systems and financial issues was first pointed to understand stock's market behavior by Bachelier [59], a Poincaré disciple, in his doctoral thesis in 1900. He applied a random walk model to describe stock prices. This study of random walk was before the remarkable Einstein´s work on Brownionan behavior and became the starting approximation of natural science to social phenomena.

Along this path of applying natural science as physics to economy, the remarkable contribution of Jan Tinbergen, who was formed as physicist obtaining his PhD in physics in 1929 and was the first laureate of the Nobel Memorial Prize in Economics in 1969, with his outstanding Gravity Model of trade mimicking Newton´s Gravitation Law for international trade flows. It should be mentioned that this work stated a basis for a trading Complex Network development [60]. Although the term econophysics doesn´t refer literally to application of Physics Laws but to the methods used in Statistical Physics (Statistical Mechanics) to understand and analyze economic

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complex systems properties constituted by a great number of human agents [61]. In addition, econophysics refers mainly to the study of macro-patterns obtained from empirical data, which is more accessible since 1990´s when computers performance was improved and is more associated to the econometric fields than to the narrative of the General Equilibrium Neoclassical Economy models with its non-heterogeneous rational-agents.

A review about the emergence of econophysics and its evolution within the Santa Fe Institute (SFI) of the hand of Brian Arthur and its further and current development can be verified on [15], [62], [44]. As mentioned, econophysics began mainly analyzing phenomena of the stock markets and their derivatives. Sociologist John Angle [63], inspired by the kinetic theory of gases, developed an approach where agents transferred money between them, analogous to collision between two particles in a gas. This work triggered the emergence of many models that analyzed the curve of the distribution of wealth by adapting models of statistical mechanics, considering for the first time the micro-processes to explain the macro-phenomena emphasizing the behavior obtained from sources with a large number of data. Currently in econophysics appear three different approaches, statistical econophysics, agent-based modeling from a Bottom-Up perspective and agent-based modeling from a Top-Down view [44]. The statistical approach analogous to statistical mechanics in the application of large particle assemblies to the regularities, present in the economic models, observed that the Boltzman-Gibbs distribution or even the Gamma distribution fitted the wide range of the distribution tails of wealth or income in general. With these models it was proved that the application of the tools of the statistical mechanics has served to the mathematization of the economy [64]. But despite the good results obtained from econophysics in the study of the distribution of wealth, it should be noted that, when applying this analogy of particle assembly to economic agents, or the concept of money to energy, a real economic system it is an open system, and in statistical mechanics they are based on closed systems.

There were three outstanding pioneering works in adopting the ideal gas model in which each agent represents a gas molecule trading money in an elastic collision, Bouchaud and Mézard [65]; Chakraborti and Chakrabarti (2000) [66]; y el de Drăgolescu y Yakovenko, [67]. The model considers a closed system, the total amount of energy or money as well as the total number of agents is conserved, where the average amount of money per agent is equivalent to energy and temperature in a system in equilibrium, obtaining a distribution of Gibbs energy or stationary Gaussian. In the work presented by [66] the concept of Saving propensity is introduced for each agent thus obtaining a Pareto's distribution, although these models refer to the concept of money rather than to the concept of material wealth. A detailed reference of the historical process, the analysis of empirical data and different models of wealth distribution is presented in [16]. In these models known as Kinetics Exchange Models the agents are defined

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through the state of the money $\{m_i\}, i = 1, 2, \ldots, N$ that each one has (Figure 6). The system evolves to a statistical equilibrium through a simple Exchange rule and algorithm as:

$$m_i^{'} = m_i - \Delta m$$

$$m_j^{'} = m_j + \Delta m \qquad \ldots \; (4)$$

where an amount of money *Δm* is transferred between randomly chosen agents *i* and *j*. *E*quation *(4)* can be varied with addition of parameters or any boundary constrains, obtaining a Gibbs like Distribution or a Log-Normal Distribution, even a Gamma like distribution as in [66]. The [67] model considers money exchange parameter $\Delta m = 1 \; with \; m_i > 0 \; and \; m_j > 0$, obtaining a stationary exponential distribution of money:

$$P(m) = Cexp\left(-\frac{m}{T_m}\right) \quad \ldots (5)$$

as in the Boltzmann-Gibbs thermodynamics gas like model of energy distribution, C is a normalizing constant, and T_m corresponds to the average amount of money per agent $T_m = \langle m \rangle = \frac{M}{N}$ (M is the total money, and N is the number of agents). Equation (5) is known as the Boltzmann-Gibbs Distribution in the econophysics literature. In Drăgolescu and Yakovenko's work variations on Δm where performed resulting the same Boltzmann-Gibbs distribution. This result was verified for medium and low income region by [33], [68], [69], [27], for an extended review of these models see [61]. Even more, in these mentioned works it is argued that a Gamma a distribution fits for the whole range of incomes. Also a log-normal distribution was applied to fit in the lower income range [8], [70], [71], [72], [73]. In [74] mentioned the analogous behaviors between socio-economic systems and a rarefied gases system which is explained through the classical kinetic theory obtaining Pareto law. They also refer to several results in economic systems obtaining similar power law decay, including for a multi-agent society in equilibrium a wealth distribution with corresponding density function F(w), for $w \gg 1$:

$$1 - F(w) = \int_w^{\infty} f(v)dv \cong w^{-p} \quad \ldots (6)$$

with p or μ (as described above) [64]. Latterly, about statistical methods Clementi et.al, [75] incorporated a κ-generalized model which fits over the whole range of incomes including upper power tail, this P(x) distribution states:

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$$P(x) = \frac{\alpha\beta x^{\alpha-1} exp_{\kappa}(-\beta x^{\alpha})}{\sqrt{1+\beta^2\kappa^2 x^{2\alpha}}} \quad \dots (7.1)$$

With cumulative $P_{>}(x)$ function as:

$$P_{>}(x) = exp_{\kappa}(-\beta x^{\alpha}) \quad \dots (7.2)$$

Radical parameter in 7.1 is obtained within Einstein's special relative framework, analogous as a relative particle system with deformation parameter exp_{κ}. In statistical econophysics topics, stochastics processes are also considered as Bouchaud-Mézard model (BM) where total wealth can change over time describing wealth redistribution as a flow based on an stochastic equation known as the Lokta-Volterra Generalized model (GLV) [76], considering wealth w_i of agent i changes over time ruled by a differential stochastic equation with exchange between agents and a random trading condition:

$$\frac{dw_i}{dt} = \eta_i(t)w_i + \sum_{j(\neq i)} J_{ij}w_j - \sum_{j(\neq i)} J_{ji}w_i \quad \dots (8)$$

where $\eta_i(t)w_i$ is a Gaussian multiplicative process that simulates an investment dynamic, and the last two terms describe the trade interaction network between the agent i and all other agents in the society. Terms J_{ij} and J_{ji} are the exchange rate between agent i and agent j. Bouchaud model expressed in equation (8) drive to other approach to study wealth and income distributions, network modelling, reviewed in section 5. The review by [16] many remarkable works dealing with econophysics applied to wealth distribution are presented. For more insight for statistical facts in wealth and income distribution refer to [77].

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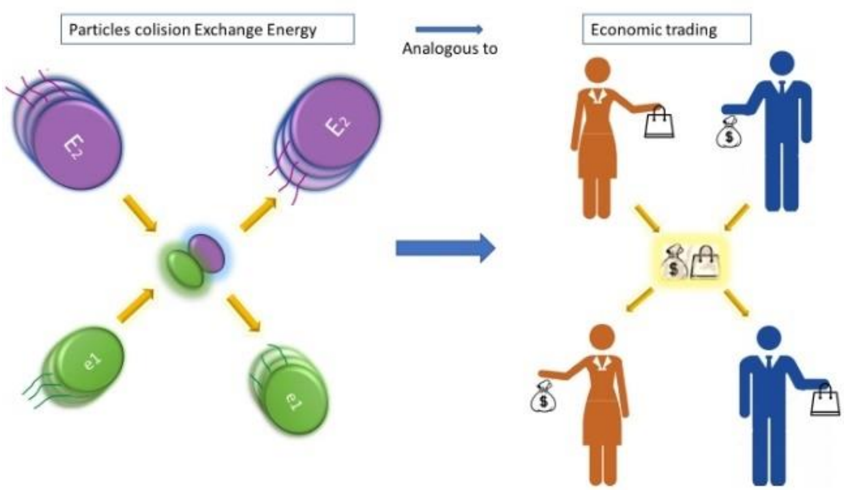

Figure 6: As in particle collisions Energy is conserved (left), in an economic Exchange money is conserved as well (right).

4.2 Agent-Based models

So far, a statistical mechanics approach has been discussed to derive size of wealth and income distribution through construction of mathematical models fitting corresponding results. No economic assumption was assumed, nor any economic paradigm was introduced. Another approach arises when applying economic theory concepts to statistical mechanics methods to model a micro-approach of agent's behavior in an economic system. Such methodologies are agent-based models. This modelling appeared in the 1990´s [78] and is applied to many interdisciplinary approaches in many different fields that is impossible to number them [79], going from economic modelling to social science simulation. Referring to an economic/financial framework dealing with power laws are discussed in this section. The development of economics, by means of applying other social science approaches to general economic fields, derived in the integration of four branches to economics framework: behavioral economics, neuroeconomics, experimental economics and agent-based computational economics (All of them led to the award of a Nobel Prize) [80]. From these we will discuss about agent-based computational economics and its application to wealth and income distribution modelling. Leigh Tesfatsion described Agent-Based Computational Economics (ACE) as a computational study of economies working with interacting autonomous learning and adaptive agents in evolving systems from a bottom-up based structure in order to reproduce regularities observed in

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economic systems resulting mathematically analogous to reaction or diffusion models presented in physics [81], [82], [83]. The same author in collaboration with Robert Axelrod have developed a web resource to provide a complete guide on ACE and a methodology applied to social science agent-based modelling (www2.econ.iastate.edu/tesfatsi/abmread.htm). Agent-Based modeling traditionally corresponds to a "bottom-up" methodology Delli Gatti, et. al, (Delli Gatti, Desiderio, Gaffeo, Cirillo, & Gallegati, 2011), published a work as an introduction to this methodology. This work is aimed to find different approaches applied to study power tails (as a Stylized Facts) in income and wealth distribution, opposite to the econophysics statistical approach, agents learning behavior create the complex structures leading to no "final equilibrium" as a difference with economic mainstream. Indeed, main objective of this approach is to reproduce the phenomenon providing a framework for these macro-patterns by giving them micro-foundations as Keynes attempted to do, considering human behavior and not only to give a statistical description [79] and [84]. In the work edited by Abergel et.al, [56], as a good reference for econophysics agent-based modelling, it is observable that this modelling approach has two important branches: one dealing with the spontaneous emergence of macro-properties without an a-priori information only through definitions of plausible assumptions. The other branch concerns on reproducing given data, and prior a calibration of assumed micro-interactions, macro-patterns are traying to be obtained, this technique is usually called "top-down agent-based modelling", for references of different published works on this methods [44] provides a good analysis. Both categories include an algorithmic rule in terms of "physically plausible" behavior. Agent-Based modeling, within econophysics framework has developed markets models, game theory and minority problems, related to microstructure, and the Kinetic Exchange Model. About this last modelling issue Chakrabarti et. al, [77] published a work focused mainly in kinetic exchange models through agent-based modelling obtaining the observed economic inequalities between different income regions. The work edited by [16] in 2005 corresponds to a Proceedings Volume of the workshop "Econophysics of Wealth Distribution"[1]. The results presented in this book are divided in two main fields which have been mentioned in this essay: data analysis and modelling. The second one corresponding to Agent-Based model paradigm proposing different models of capital exchange among economic agents trying to obtain the power law distribution for the wealthiest strata. It is worth mentioning that Farmer and Foley [3] argued that this modeling paradigm can include a realistic behavior and it has been well developed in economic. They suggest economic should shift its methodologies to improve policy makers decisions under agent-based modelling. Among these essays on wealth and income distribution another paradigm can be noticed as in the work edited by [78] where Souma, Fujiwara and Aoyama showed that in a multi-agent system if the network system

[1] This workshop was held in Kolkata, during 15-19 of March 2005, and brought together many economist and physicists in the first ever conference on Econophysics of Wealth Distribution.

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structure is considered a complex network topology, as scale free structure, emerge when agents interact each other exchanging wealth or money. These different network models will be discussed in next section.

5. COMPLEX NETWORKS

Bouchaud suggested the necessity of a new shift for alternatives to mainstream economic and financial models. In the same way Catanzaro [4] suggested that Network theory should be useful to bring this new revolution to financial and economic knowledge. So far, we have presented a brief essay of other disciplines moving into economic issues trying to explain income distribution stylized fact. As mentioned in section 3 about complex systems theories as an alternative to contribute much knowledge to orthodox economic models. Different methodologies are implemented to study complexity, in this work we considered agent-based modeling, econophysics approaches and in this section, we will discuss about complex network methodologies applied to wealth distribution. Due to current global economic inter-dependencies of both behavior and information, occurring in a world scale with every second trading of products investments credits, research, innovation the complexity involved in this global economic system is difficult to control or predict or even to study and is a natural research issue to be dealt by means of network analysis, as it is straightforward to think of agents as nodes and money, wealth or any monetary exchange as links in an economic network [85]. Network modeling considers the economic system as elements and connections where its behaviors correspond to utility performances (wealth or money exchange, trading rules, etc.), and it is not necessary to consider a production function as in mainstream economic theories. And about the systems structure evolution at all system levels, network topologies arise. Fosters [86] shows an evolution from economic simplistic traditional models to complex economic networks paradigm to represent an economic complex system. In this same path latterly Gräbner [87] contribute with an analysis for different examples for socio-economic systems that are able to be treated with Network theories. They also provide an adequate review of Network Theories evolution. This knowledge field, motivated several papers dealing with economic or financial systems treated as networks, D. McFadzean, D. Stewart, L. Tesfatsion, [88] developed what they called a computational laboratory to study evolutionary trade network formation among economic participants (buyers, sellers, consumers and dealers). Networks are the main subject of a rapidly growing literature since it includes conceptual and analytical tools already developed in sociology, computer science and physics applied to economics phenomena. The growing number of papers published concerning economic networks makes it difficult to mention not even a few of them [89]. When mentioning complex networks, it is necessary to recall prominent Erdös and Rény [90] work on random graphs which stablished the origin of this framework. Latterly experiment of Milgram in1967 suggested that the degree of separation between the people in

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United States was around [91]. At the end of 1990´s Watts and Strogratz introduced the formal modeling for the mentioned Small World Network [20]. Souma, Fujiwara and Aoyama [92] first suggested that thorough BM model a small world network emerges and varying the number of links wealth obeyed power law or log-normal distribution. Around the same years Barabási et. al, [18], published a paper introducing different phenomena behaving as scale free networks. The work presented by Albert and Barabási [93] gave a better insight on the mathematical formalism of the rising complex networks approach. Barabási published in 2002 a work showing many examples that behaves as complex scale free networks. A reference on scale free networks is [19] as well. Souma et.al, also prepared a paper mentioning how wealth distribution behaves on scale free networks using BM model [94]. In section 4.1, the BM model was presented in equation (8) where its two-last right-hand-side terms correspond to the interaction matrix (exchange matrix) within a trading agents network. This model suggests a direct approximation to complex network framework retrieving power law tails with exponents which are related to the network properties. In many papers considering this field, the BM model is applied to construct a wealth distribution model through agent's money exchange. In some of this works wealth distribution is shaped directly by the degree distribution of the network agents (nodes) as in Ma, Holden and Serota [95] showed that in the mean-field limit of a network with any two agents linked, the wealth distribution reduces to the inverse Gamma distribution. Di Matteo et. al, [96] through a wealth exchange agents system and using a stochastic BM model obtained a good fitting with Australia's income data and agreed with both the high and low-income regions. Using same BM model Garlaschelli and Lofredo [97] explore other complex network's higher order metrics such as assortativity because the node degree does not provide enough information to characterize network stationary state. They also noticed as Souma et. al, that the log-normal or the multi-mentioned power law distributions don't appear mixed as in empirical data if the network displays a homogeneous density of links, but they do if density is heterogeneous. Pei et. al [98], searched for another strategy for modeling wealth distribution in a scale free network, they applied different consumption strategies for agents' dynamic for a BM model obtaining the power law for income distribution. Ichinomiya [99] incorporated numerical simulations to the BM model including adiabatic and independent assumptions, to fit his results with observed data simulated and theory. As far, the BM model has been applied to network interactions, leading to satisfactory result on fitting simulations to power law curve. Other models also were used, as Zhang et.al(Zhang et al., 2012), applied a wealth distribution model made by Michael Gizzi, Tom Lairson and Richard Vail and using Netlogo software to simulate wealth exchange in the "sugar cape" model included in Netlogo's library. Power law was fitted. Other complex networks are also studied for wealth distribution as Vázquez-Montejo et. al(Vázquez-Montejo et al., 2010), studied wealth condensation in a Barabási-Albert scale-free network with poor-rich game exchange algorithm finding that wealth is accumulated strongly by the richest nodes. Hu

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et. al [100], examined wealth distribution on a complex network with nodes, agents, playing a Prisioners' Dilemma for exchange rules. With this model they found that for high-income group social network is scale-free, whereas it is more like a random graph for a low-income group. Lara de Paz et.al, [101] modeled 2016 Mexico's income distribution with a money exchange algorithm including a tax tribute in a Small-World network, finding that lowering taxes leads to a decrease in Gini's coefficient. Rendón de la Torre and colleagues [102] constructed Estonian Banks transaction network and studied its topological structures finding that this network, a complex network, forms a scale free network with nodes degree fitting a power law, as expected. Another approach on complex networks related to wealth distribution is the one held by Hidalgo and Hausmann [103]. They have constructed a bipartite network between countries and industrial product based on Industrial Standards (Figure 7), showing the specialization of a country. The relation between product produced by a country and different international producer for the same product is proposed as a complexity index associated to a country´s development and income distribution inequality.

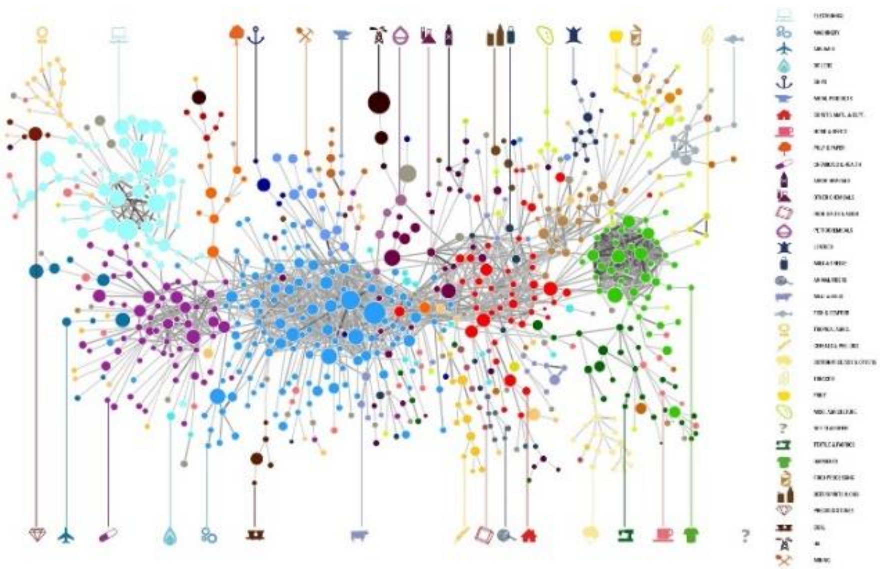

Figure 7: Mexico's Network representation of production space in the year 2015 as represented in the "Atlas de la complejidad económica". The more connected product reveals more complexity and the peripheral nodes indicate less complexity and therefore less added value. The methodology to construct the network and the Economic Complex Index is based on [103], Source:https://www.gob.mx/productividad/articulos/atlas-de-complejidad-economica-de-mexico-14291?idiom=es, Accessed 4th December, 2018.

5. CONCLUSION

In this work we made a review of some different approaches applied to the study of wealth and income distribution. Highlighting the power law as a stylized fact considered in traditional economic models. These models were pointed in their efficacy to anticipate a crisis or in considering realistic interactions between agents. Afterwards a brief description of complex system theories was mentioned and what is here to be considered is the difficult to control or describe the complexity of a system. A further analysis should consider other methodologies that use Big Data or Data mining approaches that combined with above mentioned approaches would enrich economic framework. The aim of this paper is to help policy-makers to design new tools for decision making, but the question is why these robust frameworks aren't full accepted by policy-makers or by economists? Econophysics Agent-based models should validate their results in an emergent real situation, and these models should incorporate more standard utility maximization as in traditional economic framework. Unlike complex network approach which is being e applied as showed with the development of the Economic Complex Index which is nowadays used to make a diagnosis of present structural economic state in many economic systems (https://atlas.media.mit.edu/es/rankings/country/eci/) improving decision making.

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