International Journal of Social Science and Economic Research

ISSN: 2455-8834

Volume:02, Issue:02

THE PROBLEM SOLVING PROCESSES IN MATHEMATICS

Mbuthia Ngunjiri

Senior Lecturer, School of Education, Laikipia University, Kenya.

ABSTRACT

A widely held goal of mathematics instruction is the improvement of students problem solving performance. These are many variables that may affect the problem solving performance. They include the problem, the problem solver, the problem solving processes, and the problem solving environment. In mathematics instruction, students are largely unaware of the processes involved in problem solving and addressing this issue is important. This paper focuses on mental processes involved in mathematical problem solving.

Keywords: Mathematics, Problem Solving, Problem Solving Processes

INTRODUCTION

A major challenge facing teachers, is to bring about a common body of learning in a class of students with a wide range of characteristics. These characteristics could be intellectual or emotional development, level of motivation, attitude, prior knowledge, interest, cognitive style, level of anxiety, preferred learning style, home conditions, personality and culture. Regardless of the individual differences, understanding the subject matter is an essential part of the process of learning. To facilitate understanding is the core of teaching, and in doing so the teacher must not only align the presentation of the subject matter to the logic of the content itself, but also to the level of the learners' understanding.

A widely held goal of mathematics teaching is the improvement of students' problem solving performance, in particular, explaining, treating and discussing mathematical problems takes most of the time of mathematical instruction. Learning to solve problems can be regarded as the principle reason of studying mathematics. That is, problem solving lies at the heart of doing mathematics. According to Kantowski (1997), an individual is said to be "faced with a problem when he encounters a question he cannot answer or a situation he is unable to resolve using the knowledge available to him" (p. 163). He or she must think of a way to use the information at his disposal to arrive at the goal, that is, the solution of the problem. The point is that the individual must be aware of the existence of a situation that needs a solution and must have an interest in

ISSN: 2455-8834

Volume:02, Issue:02

finding a solution. The procedure for determining the solution must not be readily available, and lastly the learner must reflect on the problem in order to develop a clear understanding of what the problem is about and how to go about finding a solution. Thus in order for a situation to be a problem for an individual, he or she must; (1) be aware of the situation, (2) be interested in resolving the situation, (3) be unable to proceed directly to a solution, and (4) make a deliberate attempt to find a solution. The following literature focuses more on the processes involved in mathematical problem solving.

REVIEW OF THE LITERATURE

The process of problem solving involves the coordination of knowledge, previous experience, and various analytical and visual abilities. Polya (2014,1973) has suggested a four step procedure for solving problems. The four phase are: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back. In the first phase, the problem-solver must see what is given, what is the unknown, and what operations are allowed. In the second phase, the problem-solver must determine a general course of attack, and in the third phase he or she must carry out the computations and other needed operations. In the fourth phase, the problem-solver looks over the steps he or she went through, trying to see how his experience can be helpful in solving other problems. In my view, the Polya's model is valuable as a guide in organizing instruction, but it is not helpful in specifying the mental processes involved in successful problem solving, or the reason as to why people differ in their performance on mathematical problems.

According to Mayer, Larkin and Kadane (1984), "peoples' performance differ because people possess information processing systems that differ and because people possess differing amounts of knowledge" (p. 233). They have argued that mathematics ability can be expressed in terms of: (1) structural and operating characteristics of the information processing systems such as the size of the working memory, and (2) knowledge contained in long-term memory. The same view is expressed by Feldhusen and Gathrie (1979), that is, they see the role of memory as crucial aspect of problem solving, and that "short-term memory serves various holding functions during work on a problem, and long-term memory are called upon for information and to serve as storage in more complex problems" (p. 29). Kantowski (1977) had argued for the limited capacity of the short term memory to deal with individual pieces of information (including chunks). As an example, in geometry, some basic theorems maybe necessary prerequisite knowledge to be held in long term memory before proving complex theorems.

Mayer et al. (1984) suggest that all problem solving is based on knowledge, and the logic of their argument is that a person may not have learnt exactly what to do in a specific problem solving situation, but whatever the person is able to do requires some knowledge. Further, they argued

ISSN: 2455-8834

Volume:02, Issue:02

that there are four steps required in solving a problem. These are: (1) translating the problem into an internal organization, (2) understanding the problem by organizing the internal representation into coherent structure, (3) developing a plan for how to generate an answer, and (4) executing the problems using relevant algorithms. The four steps require different types of knowledge in a problem-solver. For the translation phase a person needs linguistic and factual knowledge, and for the understanding phase the person needs schematic knowledge, for the planning phase the person needs strategic knowledge and for the execution phase the person needs algorithmic knowledge. As an example, suppose the problem is to find two consecutive numbers whose product is fifty six, the source of individual differences in solving the problem may be due to differences in domain specific knowledge. An individual may be unable to generate the answer due to lack of linguistic or factual knowledge such as translating the problem words into the corresponding equation or knowing the meaning of the word consecutive. An individual may be unable to get the correct answer due to lack of schematic knowledge, that is, knowing the problem type or form, which is quadratic for the particular example. Another source of inability maybe due to lack of strategic knowledge, that is, the individual must know when to use certain operations such as multiplication, addition, finding roots and so on. Lastly, failure to solve the problem may be due to lack of appropriate algorithms such as how to multiply or add negative and positive numbers.

However, there are other variables that may affect an individual's ability to solve a problem. Mayer (2008, 1975) has suggested three models of internal information processing involved in learning to solve problems. The one-stage model is based on the idea that there is one critical condition of learning, that is, "how much of the presented material is received by the learner" (Mayer, 1975, p. 526). According to the one-stage model, the more information which is presented or the slower the presentation or the more practice allowed or the stronger the instructions to pay attention, the more likely the subject can pay attention to and receive the material.

The two-stage model states that there are two critical conditions, that is, "receiving the information and having appropriate prerequisite knowledge in memory" (Mayer, 1975, p. 526). The point in the model is that the main internal processing variables are both the learner's ability to pay attention to new material and the amount of prior knowledge already in the learner's memory at the time of learning. The new material so received is added to memory.

The three-stage model considers not only how much information is received, and how much prerequisite knowledge the learner has, but also those aspects of the learner's existing knowledge that are activated with the new material. The point in this last model is that information from the outside world enters the learner through sense receptors and eventually reaches active

ISSN: 2455-8834

Volume:02, Issue:02

consciousness, at the same time the learner may possess existing knowledge, and may search and bring some of this existing knowledge to active consciousness. Mayer (2008, 1975) further has argued that the three-stage model proposes: (1) that outcome of learning be evaluated in both how much is learned and how is learning structured, (2) that the main processing variables are reception of the material, existence of prerequisite knowledge, and activation of assimilative set, and (3) that the external instructional variables influencing activation and integration of new material may be sequencing and emphasis of instruction. He also argued that the acquisition of new information in long-term memory be represented as adding new nodes to memory, and internally or externally connecting the new nodes with aspects of the existing network. Internal connectedness refers to the degree to which new nodes are connected with one another into a well defined single structure, while the degree to which new nodes are connected with concepts already existing in the learner is the external connectedness.

The point in Mayer's model(2008,1975) is that: (1) if the learner has the general prerequisite knowledge and has actively searched for how to use part of it as an assimilative set during learning but has failed to take in the specific information, the result would be an inability to answer specific questions which require specific information (2) if the learner has received the specific information and has had previous experience but have not activated the proper aspects of his cognitive structure to help integrate the material, then the result would be an ability to answer very familiar and specific questions, but cannot effectively generalize one problem to another, and (3) if the learner has received the specific information and has actively searched for the proper integration set but it is without the general prerequisite knowledge, then the learner will have difficulty with essay questions or other problems requiring him to extend the specifics of what he or she learned.

Feldhusen and Guthrie (1979) argued that cognitive style (field dependence-independence) seems to bear relationship to abilities involved in problem solving. They have stated that "when the problem-solver is limited to the perceptual field in which a problem is embedded, that person is field dependent, and when the person experiences elements of the problem situation as separate from the contextual field, that person is field independent" (p. 29). The point is that since most problem situations involve a relatively complex field of elements, then field dependence-independence will influence the problem solving behavior.

Kantowski (1977) argued that reflexivity-impulsivity has a relationship with the problem solving style. Kantowski states that reflexive students do use analyses of material too much, whereas impulsive students often do not use enough analysis to determine a plan to a solution if it was not immediately obvious. The point is, those students during the planning and execution stages of a problem who respond quickly tend to err (impulsiveness), whereas those who pause to reflect are

ISSN: 2455-8834

Volume:02, Issue:02

most often correct (reflective). Kantowski (1977) has further argued that persistence is a variable during problem solving and seems to be affected by both prior knowledge and reflexivity-impulsivity, in that "reflexive individuals tend to be more persistent on difficult tasks than impulsive students" (p. 169).

In sum, as seen from the foregoing there are general mental processes for problem solving which are cognitive in nature while others are dispositions of the problem solver, which all play a role in problem solving behavior.

CONCLUSION

There are many variables that affect problem solving performance. These variables fall into four distinct but interrelated categories, that is, the problem, the problem solver, the problem solving process, and the problem solving environment. The mathematical content of a problem is important as well as the structure of the problem, for example, linguistic and logical structures. The characteristics of the problem solver includes prior knowledge, cognitive style, perseverance and attitudes. The problem-solving process is closely related with both problem and problem-solver's variables. This may include the manner in which the problem and problem solver organizes and processes information, the types of cognitive strategies used to plan and carry out a plan and the methods used to evaluate what has been done, such as the looking back strategies. The problem solving environment, though very little of it was mentioned, is also a variable. These are those forces that are external to the problem and problem solver. These may include instruction, conditions under which the individual works such as stress, and maybe time-constraints. All these variables, though not mutually exhaustive, interact with each other throughout the course of solving a problem, and the interactive nature of these variables is what causes problem solving to be either simple or a difficult activity.

REFERENCES

Feldhusen, J. F. (1979). Models of problem solving and abilities. *Journal of Research and Development in Education,* 12 (2), 22-32.

Kantowski , M.G. (1977). Processes involved in mathematical problem solving. *Journal for Research in Mathematics Education.* 8(3), 163-180.

Mayer,R.E.;Larkin,J.H.& Kadane,J.B.(1984). A cognitive analysis of mathematical problem solving ability. In R.Sternberg(ed.),*Advances in the psychology of human intelligence*(pp.231-273).Hillsdale,N.J.: Lawrence Erlbaum Associates.

Mayer, R.E. (2008). *Learning and instruction* (2nd Ed). Boston; Little, Brown and Company.

ISSN: 2455-8834

Volume:02, Issue:02

Mayer, R.E. (1975). Information processing variables in learning to solve problems. *Review of Educational Research,* 45 (4), 525-541.

Polya, G. (2014). *How to solve it: A new aspect of mathematical method*. Princeton: Princeton University Press.

Polya,G.(1973).*How to solve it*. Princeton N.J.: Princeton University Press.

