
ESTABLISHING WHICH ARCH FAMILY MODEL COULD BEST EXPLAIN VOLATILITY OF SHORT TERM INTEREST RATES IN KENYA.

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ABSTRACT

The relationship between short-term interest rate volatility and interest rate levels has been widely documented. This study focused on establishing the connection between the level of interest and the volatility of interest rates in Kenya using data from December 1994 to December 2014. The main variable for the study was the short term interest rate series. The results of the study were consistent with the hypothesis that the volatility is positively correlated with the level of the short term interest rate as documented by previous empirical studies.

Keywords: Interest rates, Volatility, 91-day T-BILL Kenya

1.1. INTRODUCTION

The Classical theory of interest defines the rate of interest as the factor that equates savings and investment. The investment is the demand for investible resources and savings is their supply. The rate of interest that is determined by the interaction of investment and savings is the price of the investible resources. According to Marshall the interest rate is the price paid for the use of capital. This rate of interest is determined by the equilibrium formed by the interaction of the aggregate demand for capital; and its forthcoming supply. On the other hand Keynes had his own way of defining interest rates. According to him, the interest rate influences the marginal propensity to save. This savings is also linked to the level of income. Hence he concludes that the rate of interest should be at a point where the demand curve for capital at different interest rates intersects the savings curve at a fixed income level. However these theories had their own weaknesses and therefore challenged by other economists. For example the classical theory faces the following criticism, If the interest rate, the demand for capital and the sensitivity of the marginal propensity to save to a change in the interest rate are all given then the income level would be the factor that would equate savings with investment.

Banks and other micro finance institutions charge fees to their customers when they advance loans to them. When these loans are to be repaid within one year then these are referred to as short term loans and the applicable fees are what we call short term interest rates. These interests are quite useful in many economic spheres including modelling of economic models.

Studies Turan and Liuren, 2005 indicates that short term interest rate level is positively correlated with volatility. Typically these rates are affected by both monetary and fiscal policies of any given country. During inflation short term interest rates are manipulated to drive the economy towards a certain desired direction. When there is a lot of money in circulation the rates are increased to discourage further borrowing. When there is little money in circulation these rates are lowered to encourage borrowing from the commercial banks. To ease the cost of borrowing sometimes the central bank intervenes and sets interest rates caps for commercial banks. In 2015 the central bank of Kenya capped the base lending rates at 10%. This therefore means commercial banks can only charge up to 4% higher than the 10%. This was a move to protect Kenyans from exploitative commercial banks. And this how interest rates are quite important in any given economy.

1.2. Kenyan short term interest rates.

In Kenya the short term interest are the 91 day treasury bills. These rates are reported by the central bank every month and they are usually the monthly averages. In December 2016 the rates were 8.14% per annum. Since 1993 to date the rates have been fluctuating depending on the prevailing microeconomic conditions as well as the interventions by the central bank. July 1993 recorded the highest interest rate value of 84.67% pa while September 2003 recorded the lowest value at 0.83% pa.

1.3. Volatility Modelling

The original work of Engle (1982) and Bollerslev (1986) introduced that generalized autoregressive conditional heteroskedastic (GARCH) models were handy if we model the time-varying volatility of the financial assets. Therefore it became the bedrock of the dynamic volatility models, see Alexander and Lazar (2006). The advantage of these models were that they were practically easy to estimate and could allow researchers perform diagnostic tests.

However, GARCH (1, 1) only captured some of the skewness and leptokurtosis (fat tails relative to the normal distribution) in the financial data. Alexakis and Xanthakis (1995).

Bollerslev (1987), Baillie and Bollerslev (1989), Nelson et al.(1996) also found that if the observed conditional densities was non-normal, it was higher than that could be forecasted by

normal GARCH(1,1). Therefore, more researchers explored alternative distributional functions for the error term in order to supply a better explanation of data.

Consequently, numerous non-normal conditional densities had been introduced in the GARCH framework. In particular, Bollerslev (1987) presented the Student t-GARCH that had also been captured by GARCH models (Alexander and Lazar, 2006). These developments in GARCH models were obviously crucial for Modelling volatility variation.

If the conditional variance did not follow the normal distribution, the normal GARCH model could not explain the entire leptokurtosis in the sample data and it was better to apply the non-normal distributions, such as Student t, normal-lognormal distribution or the exponential GARCH model to capture higher conditional moments, see Alexander and Lazar (2006). On the other hand, many authors (Christie, 1982; and Nelson, 1991) had pointed out the evidence of asymmetric responses, suggesting the leverage effect and differential financial risk depending on the direction of price change movements.

In response to the weakness of symmetric assumption, Nelson (1991) brought out exponential GARCH (EGARCH) models with a conditional variance formulation that successfully captured asymmetric response in the conditional variance. EGARCH models had been demonstrated to be superior compare to other competing asymmetric conditional variance in many studies, see Alexander (2009).

1.3.1. Arch Models

The ARCH (q) model can be expressed as:

$$\begin{aligned}\varepsilon_t &= z_t \sigma_t \\ z_t &\sim i.i.d. D(0, 1) \\ \sigma_t^2 &= \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2,\end{aligned}$$

where $D(\cdot)$ is a probability density function with mean 0 and unit variance.

The ARCH model can describe volatility clustering. The conditional variance of ε_t is indeed an increasing function of the square of the shock that occurred in $t-1$. Consequently, if ε_{t-1} was large in absolute value, σ_t^2 and thus ε_t is expected to be large (in absolute value) as well.

Notice that even if the conditional variance of an ARCH model is time-varying

$$(\sigma_t^2 = E(\varepsilon_t^2 | \psi_{t-1})), \text{the}$$

$$\omega > 0 \text{ and } \sum_{i=1}^q \alpha_i < 1, \text{we}$$

unconditional variance of ε_t is constant and, provided that

$$\sigma^2 \equiv E(\varepsilon_t^2) = \frac{\omega}{1 - \sum_{i=1}^q \alpha_i}.$$

have We

1.3.2. Garch Models

The ARCH model was introduced by Engle (1982) and later extended by Bollerslev (1986), who developed the generalized ARCH, or GARCH model. In a GARCH (1, 1) model (equation 3), the conditional mean and conditional variance of a time series process are modelled simultaneously. It is based on an infinite ARCH specification and it allows to reduce the number of estimated parameters by imposing non-linear restrictions on them.

The GARCH (p; q) model can be expressed as:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2.$$

In a GARCH (1, 1) model the conditional mean and conditional variance of a time series process are modelled simultaneously.

$$r_t = \alpha + \beta r_{t-1} + \varepsilon_t \dots\dots\dots$$

Where the conditional volatility of ε_t is given by equation

$$E[\varepsilon_t^2 / \phi_{t-1}] = h_t \dots\dots\dots$$

and $h_t = \omega + \theta \varepsilon_t^2 + \psi h_{t-1} \dots\dots\dots$

$\alpha, \beta, \omega, \theta$ and ψ are regression constants.

r_t represents the interest rate series.

Unlike ARCH, GARCH models can show significant volatility clustering occurrences which have been documented in numerous financial time series. One of the shortcomings of GARCH models as far as short rates are concerned is that the parameter estimates shows that the volatility process is explosive where $\alpha + \beta > 1$. Gray (1996) for example found that $\alpha + \beta = 1.0303$ using weekly 30-day T-bill data.

2.1. METHODOLOGY

The short term interest rate series in Kenya is the Central Bank three-month Treasury bill rate taken from the Central Bank of Kenya Database. The data can be found on the following link <https://www.centralbank.go.ke>. The study applied the monthly averages of the 91-day TBILL rate for the period between December 1994 and December 2014. Due to market liberalisation of interest rates in 1991, only market forces determined the level of short term interest rates hence idea period for this study

Unit root tests were conducted to establish the nature of the data. The short interest rate series showed that the data was non stationery. Therefore differenced series (the series was lagged by 1 level) was applied for Modelling volatility. The decision rule was based on rejecting H_0 : the series is non-stationary, if the ADF statistics are less than the critical values (Dickey and Fuller, 1979).

ARCH test were carried out and was significant confirming the presence of ARCH effect. Since ARCH effects were present ARCH and GARCH models were run to establish the most appropriate model for estimating volatility of short term interest rates in Kenya. Diagnostic checks for ARCH effect, serial correlation and normality test were applied to the residuals to establish if the residuals significantly affected the short term interest rate model. Later ARCH LM test was done on the model to check if ARCH effects were still present.

2.2. Data and Basic Statistics.

2.2.1. Statistics

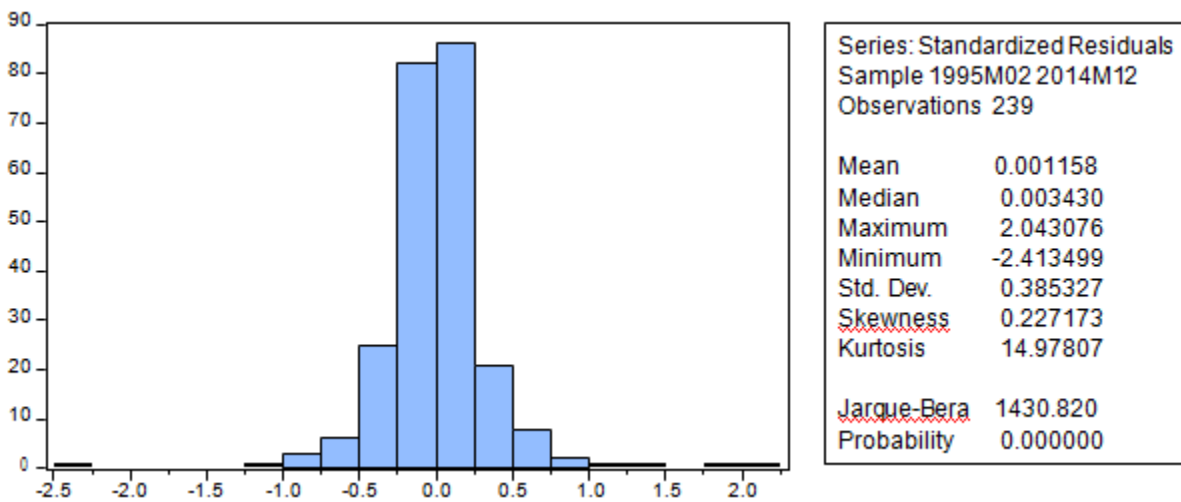


Figure 1. Histogram

Descriptive statistics are represented by the above Histogram as shown by Figure 1. The statistics shows that the short term interest rates are positively skewed meaning that the distribution is asymmetrical with a long tail to the right. Kurtosis is 14.978 a positive figure that is greater than 3 signifying the heavier tail than standard normal distribution. This means that the distribution is more peaked and has fatter tails. This is also known as leptokurtic. The JB test has also rejected the null hypothesis of normality which mean that series are not distributed normally.

2.2.2. Data Analysis

Since the short term interest rates were non stationary as shown by figure 2, the original series was transformed into stationary series as shown by figure 3 and Modelling was performed based on transformed-stationary series. A special class of non-stationary process is the I(1) process (i.e. the process possessing a unit root). An I(1) process may be transformed to a stationary one by taking first order differencing. This was achieved by employing the Augmented Dickey-Fuller (ADF) unit root tests (Dickey and Fuller, 1979) to check whether the T-BILL rates data series was stationar. The null hypothesis,

- H_0 is that r has unit roots
- H_1 is that r is integrated at order zero, $I(0)$.

The hypothesis was tested at a critical level of 1% and 5% as shown below

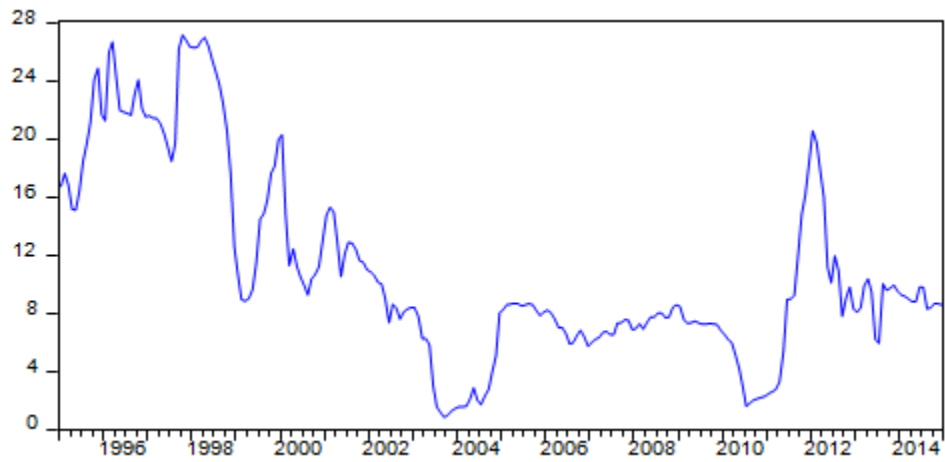


Figure 2 Short term Interest rates trend.

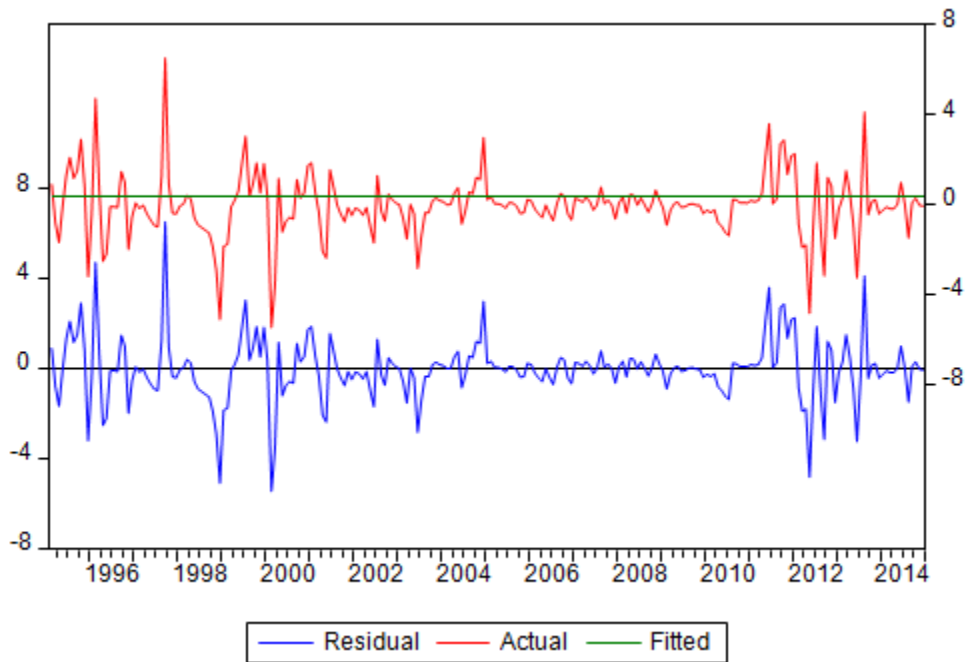


Figure 3. Differentiated Short term interest rates

Unit Root Testing

Short Term interest rates in Kenya			
Test	T Statistics at 1% and 5%	T statistics	Probability
ADF	-3.457984 & -2.873596	-2.442771	0.1312
PP	-3.45763- & -2.87344	-2.150144	0.2254
KPPS			1.084835
ARCH			0.0000
1st Difference of Short term interest rates			
ADF	-3.457984 & -2.873596	-6.792819	0.0000
PP	-3.457747 & -2.873492	-10.09741	0.0000
KPPS			0.0442

Table 1. Unit Root Testing results

From figure 3 above shows volatility of short term interest rates in Kenya. Periods of high volatilities were followed by corresponding periods of high volatility over a long period of time. Similarly low period of volatility were followed by low periods of volatility over a long period of time. As per Table 1 above ADF, PP and KPSS unit root test were done and they confirmed that the interest rate data was stationery. Its' therefore justified to run ARCH family models.

3.1. MODELLING VOLATILITY OF SHORT-TERM RATES

3.2. Arch LM Test for Level Effects and Asymmetry

The residuals of the regressions of the short term interest rates series were tested for level effects using the ARCH LM test and the results are presented in Table 2 below.

Heteroskedasticity Test: ARCH

F-statistic	39.26705	Prob. F(1,236)	0.0000
Obs*R-squared	33.95088	Prob. Chi-Square(1)	0.0000

Table 2. ARCH LM Test for Level Effects and Asymmetry

To establish whether to run ARCH family model, the Arch LM test was done on the residuals using the differenced series. The LM test was based on the null hypothesis that the differenced series had no level effects. The decision rule was based on rejecting the null hypothesis if the computed Chi-square statistics were greater than critical values of a known chi-square distribution at 95% levels of confidence. The results shows that the residuals developed for the T-BILL differenced short rate had level effects. Since the variance of the errors is not a constant, heteroscedasticity exists for the residuals of the short-term interest rate.

3.3. Modelling Volatility Using ARCH/GARCH Models

The objective of modelling the stochastic volatility underlying 91 -day T-BILL rate changes in Kenya is to allow for determination of better forecasting models by players in the Kenyan financial markets. Empirical evidence indicates that parameters for the models shift over time (Johnston and Scott, 1999), therefore it is more appropriate to calculate model parameters from time to time. Accurate descriptions of the short term distributions would allow for development of improved forecasting models. In this study, the parameters of the GARCH (1, 1) and ARCH (1, 1) models were calculated over the sample period, using maximum likelihood estimation. The findings derived of the maximum likelihood estimation are presented in Table 4 below.

Model	Coefficient	Value	Z-Statistic	P-values	Decision
ARCH (1,1)	Constant	0.6240267	1.74	0.081	Accept H ₀
	Lag (1)	0.6932527	4.77**	0.000	Reject H ₀
	Lag (2)	0.193107	1.48	0.140	Accept H ₀
	Lag (3)	-0.4378276	-2.63**	0.008	Reject H ₀
GARCH (1,1)	Constant	0.6240267	1.74	0.081	Accept H ₀
	Lag (1)	-0.1852768	-1.56	0.120	Accept H ₀
	Lag (2)	0.5983886	3.51**	0.000	Reject H ₀
	Lag (3)	0.0868504	1.21	0.227	Accept H ₀
LR Statistic = -386. 5642**					
Wald Chi-square Statistic (d.f. = 1) = 7.43E+11 **					

H₀: Value of Constants =0 vs. H₁: Otherwise

* Denotes significance at 5% critical level (P-values < 0.05)

** Denotes significance at 1% critical level (P-values < 0.01)

Table 3 . ARCH and GARCH results

The findings of Table 3 above indicate that the residuals of the two models have the volatility clustering effect and this is indicated by the significant coefficients of the ARCH(1) and GARCH(1) terms in the variance equation of the differenced 91 day Treasury bill rate. The sum of the significant coefficients on the lagged squared error and lagged conditional variance is less than one in all the cases. The sum equals 0.255426 for the ARCH (1,1) model (equivalent to lag 1 + lag 3 since lag 2 is not significant) and 0.5983886 for the GARCH (1,1) model (equivalent to lag 2 only since lag 1 & lag 3 are not significant). This sum is close to unity in the case of GARCH model indicating that shocks to the conditional variance will be highly persistent. A large sum of these coefficients implies that a large positive or a large negative return will lead future forecasts of the variance to be high for a protracted period.

Similarly Wald test was done to check if $\alpha+\beta$ (persistence coefficients) , are equal to one or not. The results as shown in Table 4 leads to rejection of null hypothesis of $H_0; \alpha+\beta=0$. This means that $\alpha+\beta \neq 1$ hence revealing persistence of the shock for short period.

3.4. ARCH Lagrange Multiplier Test for Level Effects

The residual series obtained from the estimated ARCH and GARCH models of Table 4 and table 5 respectively above were tested for level effects. Both models did not have ARCH effects. to see if level effects are captured well in the estimated model. The findings are presented in Table 5 and 6 below.

Heteroskedasticity Test: ARCH

F-statistic	0.649595	Prob. F(1,236)	0.4211
Obs*R-squared	0.653301	Prob. Chi-Square(1)	0.4189

Table 4 ARCH LM Test on ARCH (1, 1) model

Heteroskedasticity Test: ARCH

F-statistic	1.384922	Prob. F(1,236)	0.2405
Obs*R-squared	1.388510	Prob. Chi-Square(1)	0.2387

Table 5. ARCH LM Test on GARCH (1, 1) model

The findings above indicate that the ARCH effects are not present in the model estimated after taking into account the GARCH terms. Thus, the GARCH model is better than the ARCH model for modelling volatility of short-term interest rates. However, the GARCH models estimated do not take into account the leverage effect and hence the E-GARCH models can be developed to test whether asymmetric effects are present.

Diagnostic tests were further done on the models. These were ARCH LM test, serial correlation test and normality test. The models passed the serial correlation test and ARCH LM test though it failed to meet normality test. This however cannot be used to discard the model since Economists conquer that although residuals may not be normally distributed the model could still be accepted.

Therefore the study identifies that the GARCH model is better suited for Modelling volatility of short rates in Kenya, as opposed to ARCH models since its able to capture the very important volatility clustering phenomena that has been documented in many financial time series, including short-term interest rates.

4. CONCLUSION

The objective of this study was to develop a model that could best describe the behaviour of short term interest rates in Kenya. Historical data for the monthly (average) 91-day T-BILL rates which were obtained from the Central Bank of Kenya was used. The key findings revealed that there is a relationship between the level of short-term interest rates and volatility of interest rates in Kenya. Similarly it was discovered that GARCH models are perfect estimators of volatility of short rates in Kenya than ARCH models. These findings are also in line with the hypothesis that the volatility is positively correlated with the level of the short term interest rate as documented by previous empirical studies (Olan and Sandy, 2005; Turan and Liuren, 2005).

4.1. Further research

In this study the GARCH model was found to be the best estimator of the relationship between volatility and short term interest rates in Kenya. However, the model do not take into account the leverage effect and hence the E-GARCH model is suggested for further research to test whether asymmetric effects are still present.

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